What is persistence?

TDA group at KTH Stockholm

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M. Scolamiero, W. Chacholski

Persistence is

Persistence is a process of transforming discrete invariants into continuous invariants

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Discrete invariant: $f : T \to \mathbb{N}$

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A collection of data sets

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triangular inequality

symmetry

d(X,Y)=0 does not imply X=Y

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measurable functions

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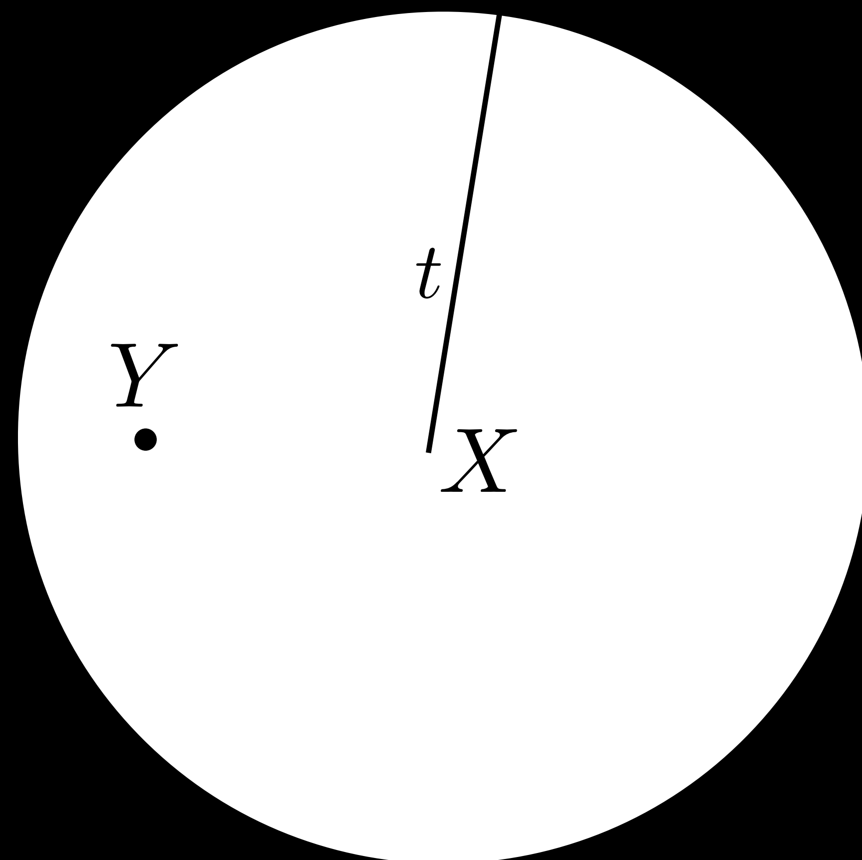
$$\hat{f}(X)(t) = f(Y)$$



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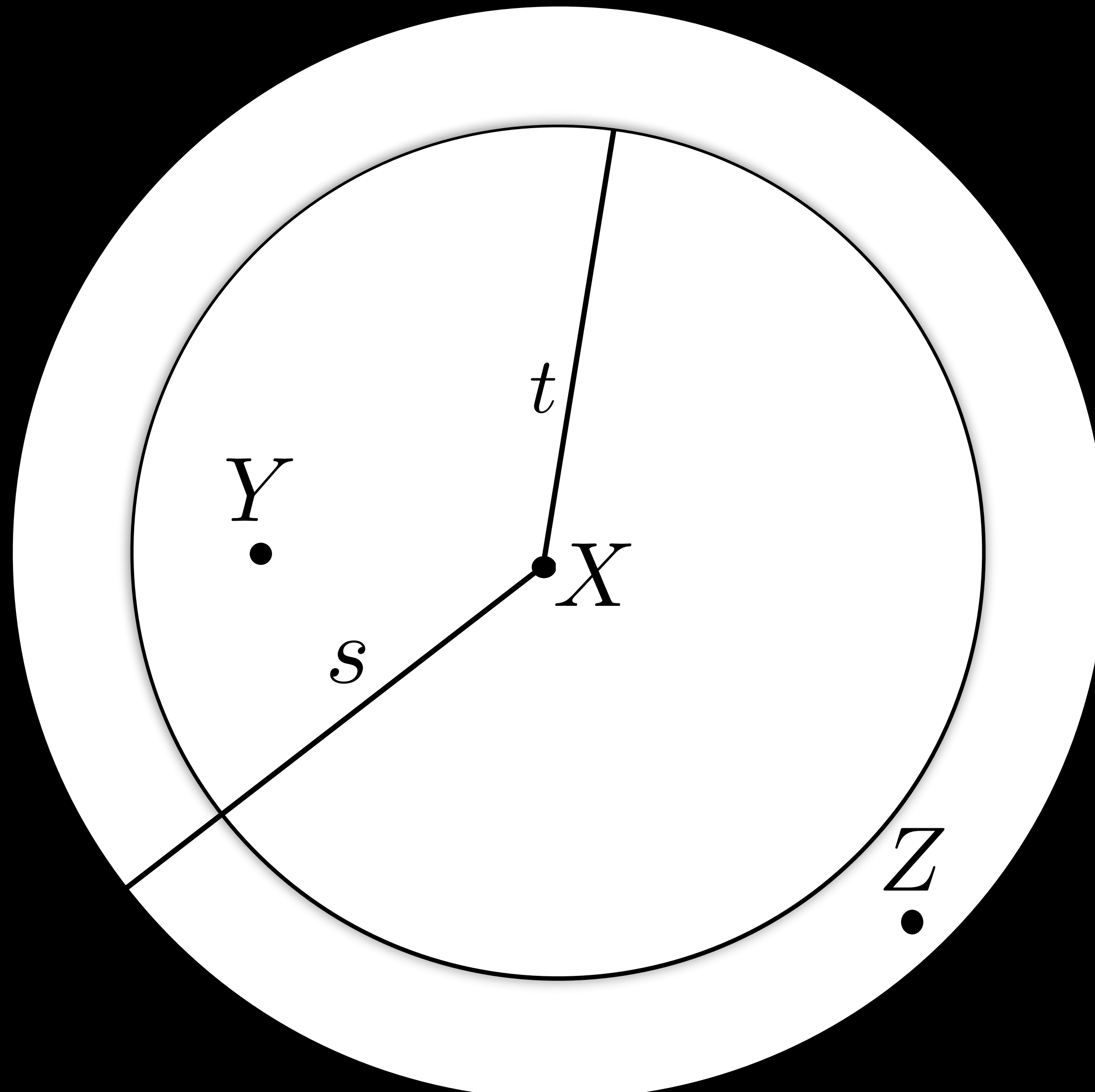

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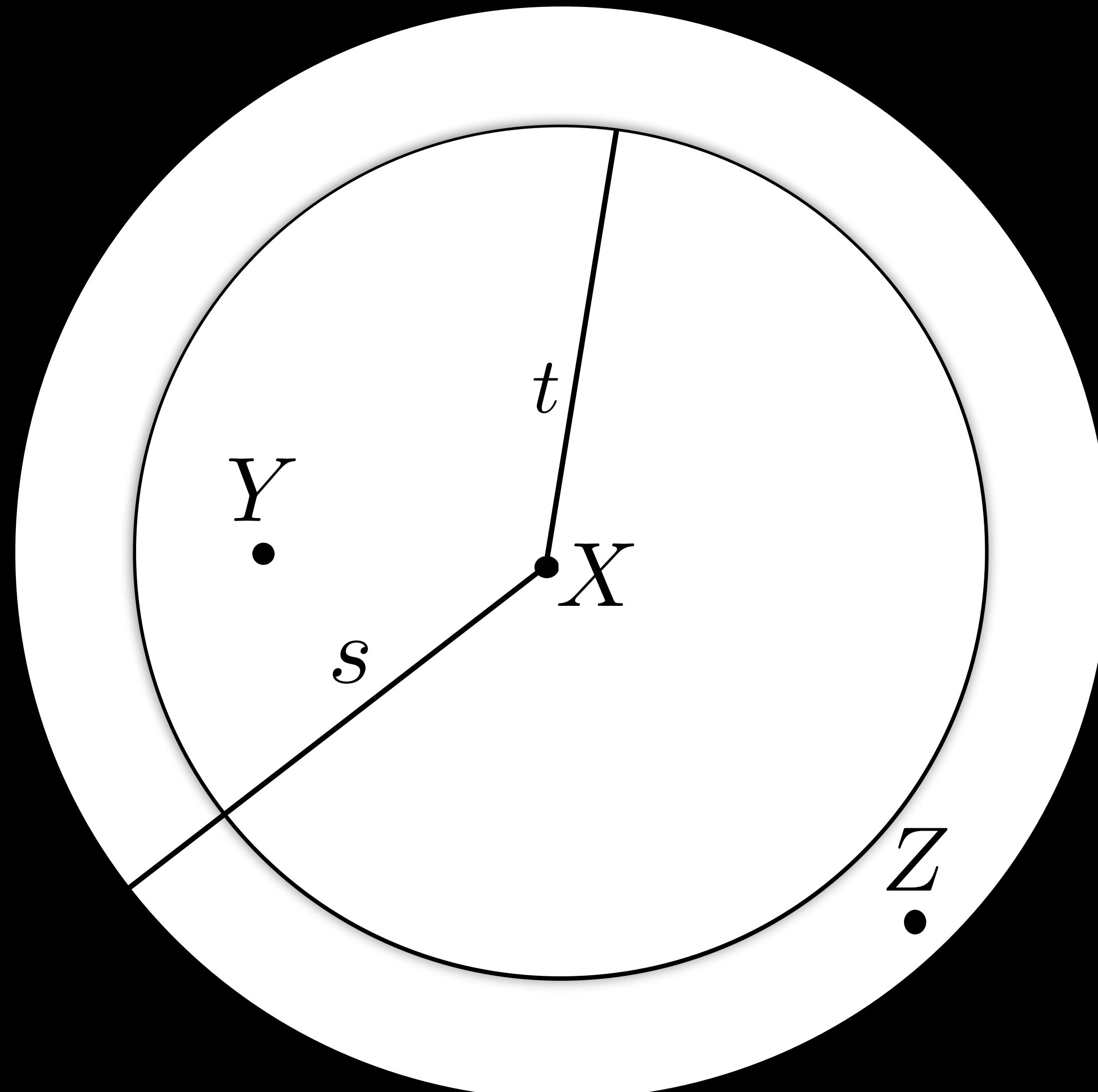

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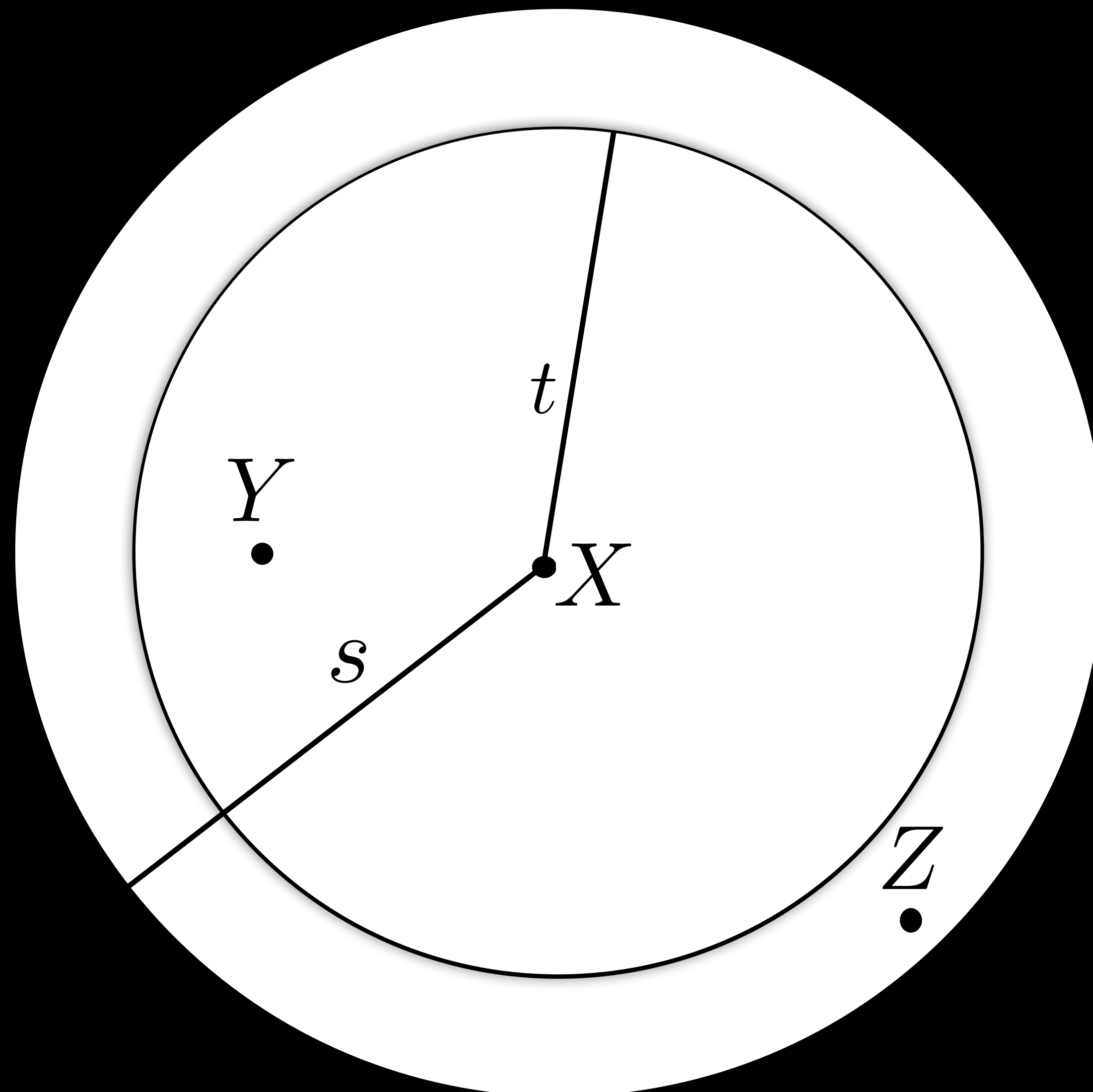

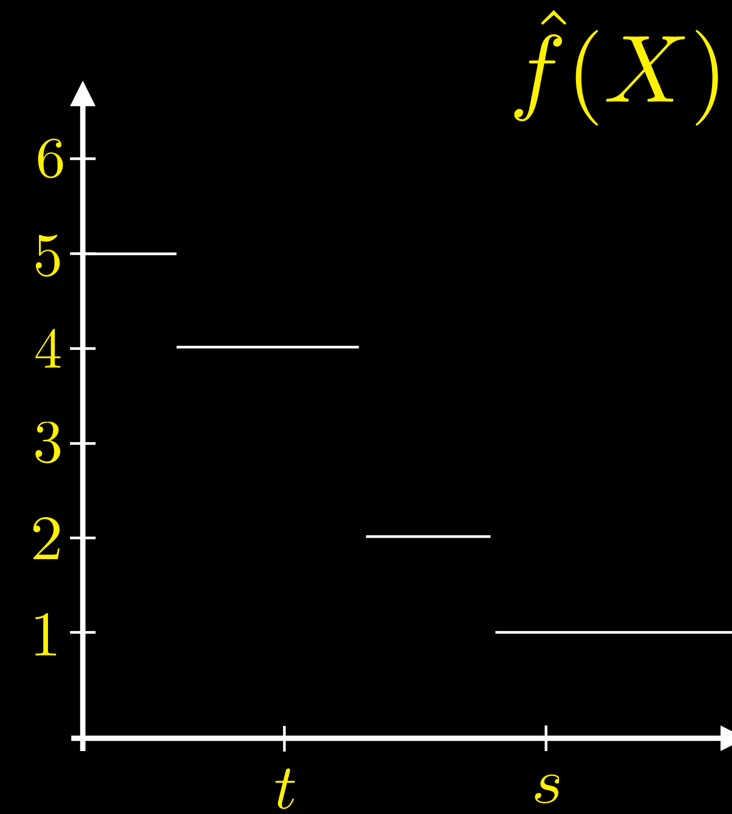

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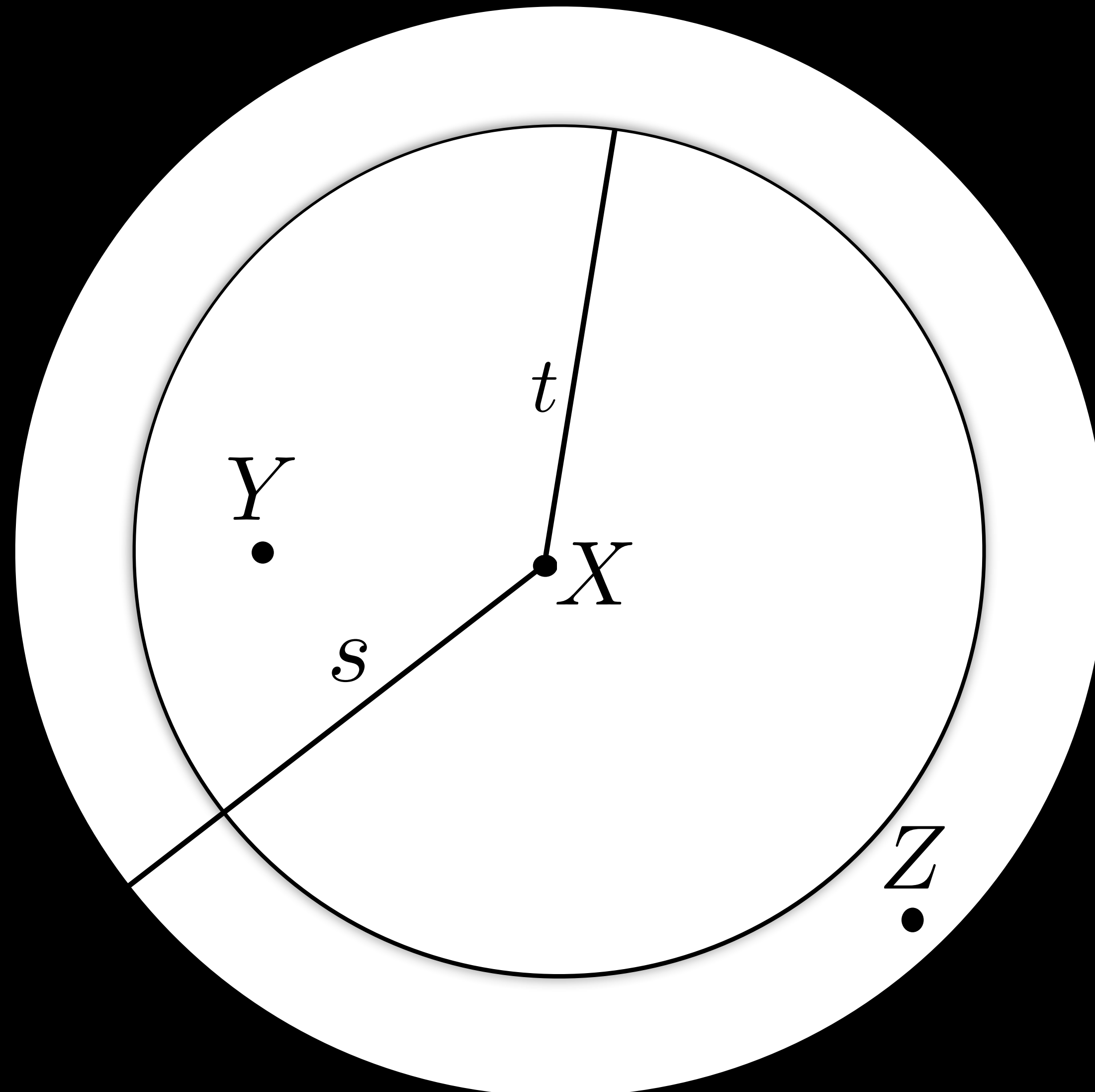

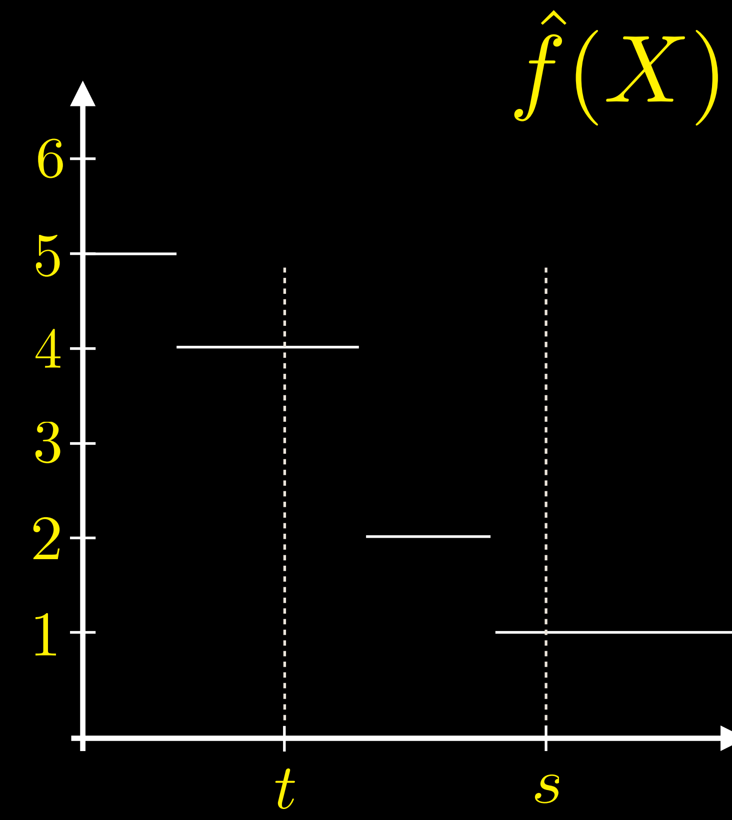

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Metrics on $\mathrm{Func}(\mathbb{R}_{\geq 0}, \mathbb{R}_{\geq 0})$

Integral:

$\int(\alpha, \beta) = \int_{t=0}^{\infty} |\alpha - \beta|$ $\qquad$ $\int_a^b(\alpha, \beta) = \int_a^b |\alpha - \beta|$

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Theorem: $\hat{f}\colon T \to \mathrm{Func}(\mathbb{R}_{\geq 0}, \mathbb{R}_{\geq 0})$ is continuous.

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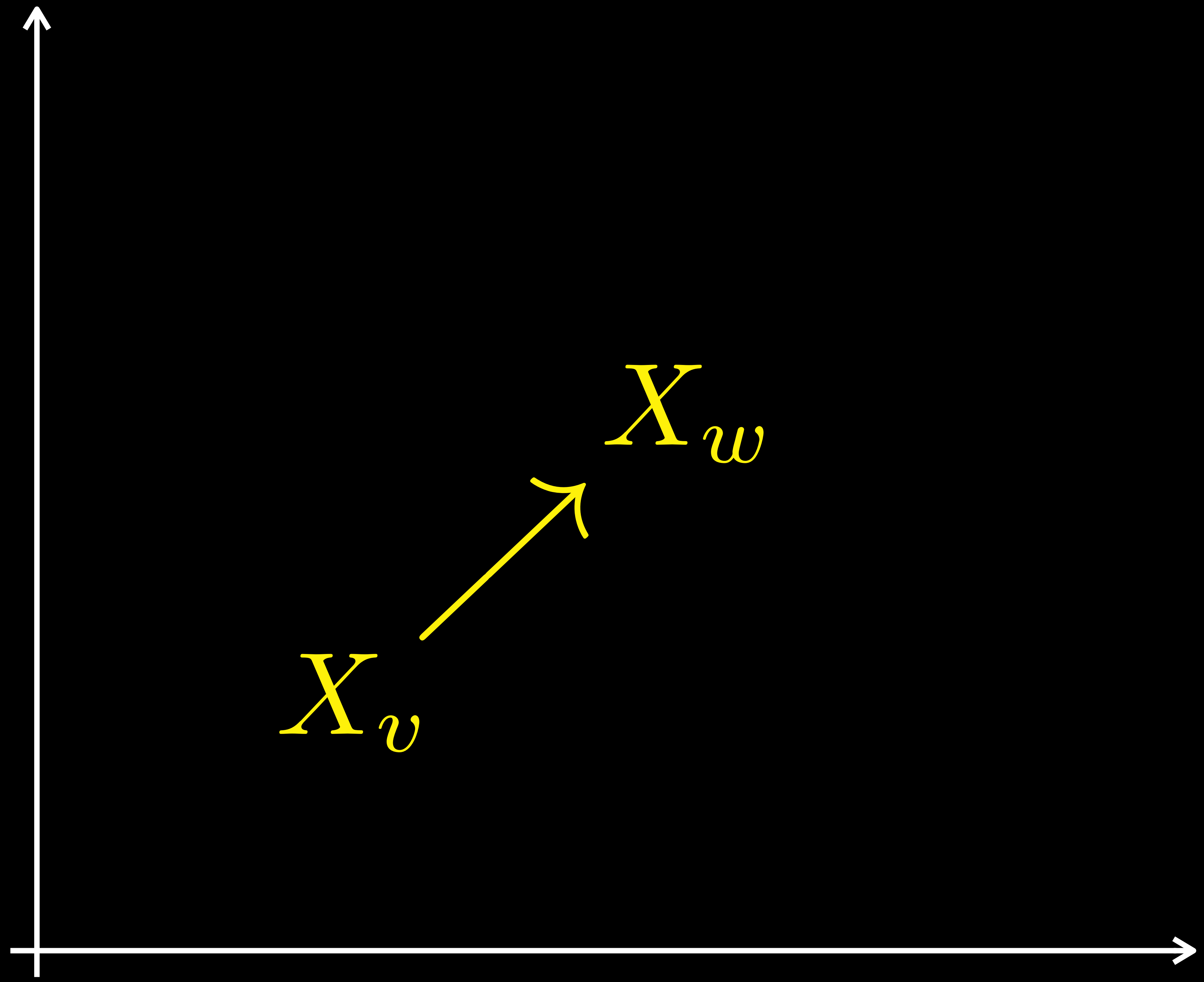
Compact Tame Vector Spaces parametrized by

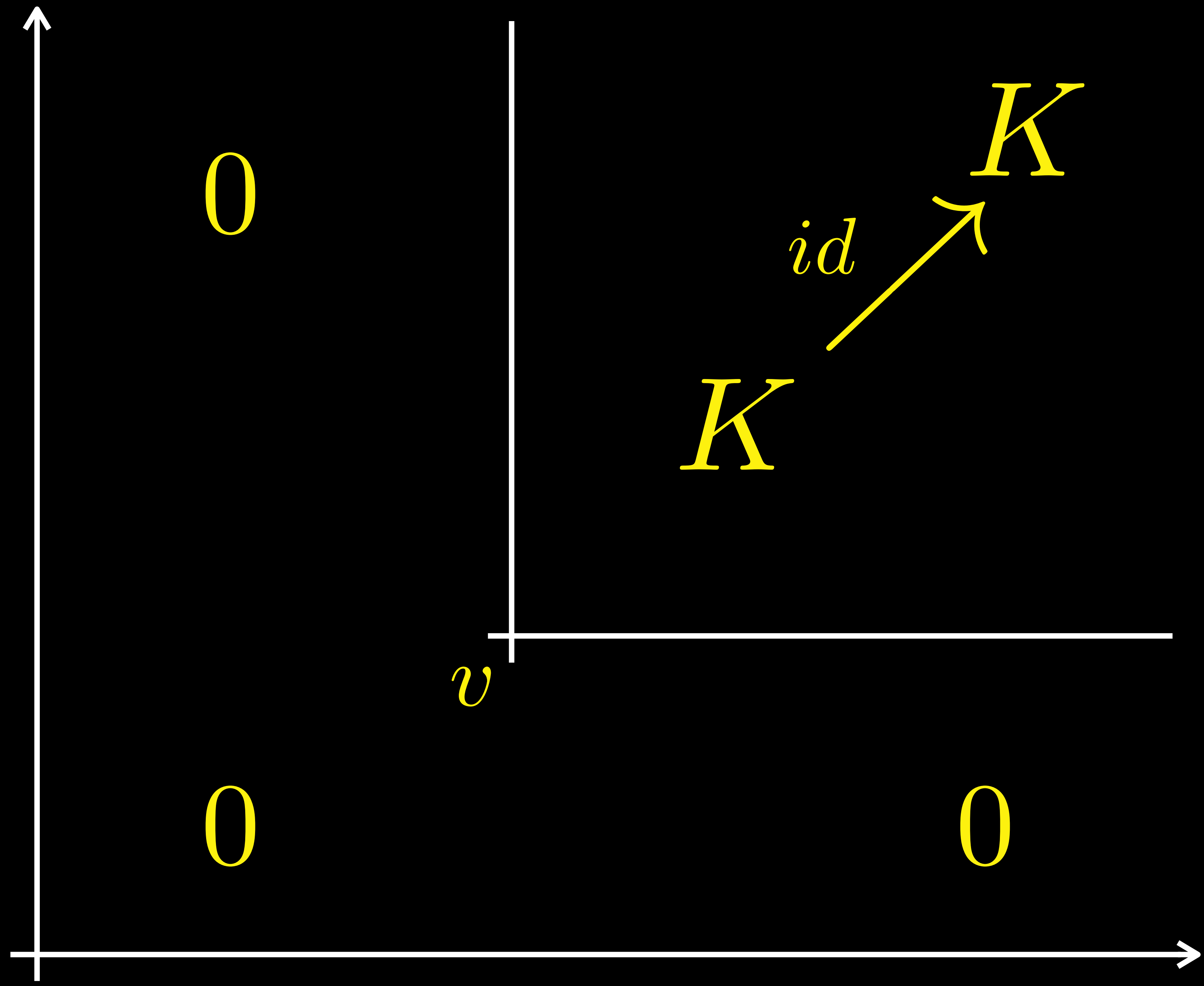
$\mathbb{R}^n_{\geq 0}$

X

$\mathbb{R}^2_{\geq 0}$



$K(v, -)$ $\mathbb{R}^2_{\geq 0}$



$\mathbb{R}^2_{\geq 0}$

X has rank k if there is a surjection:

$$\bigoplus_{i=1}^{k} K(v_i, -) \to X$$

and k is the smallest such number

T

Compact Tame Vector Spaces parametrized by

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$$T$$

Compact Tame
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Sets with n measurments

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Metrics on T compact tame $\mathbb{R}^n_{\geq 0}$ vector spaces

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↑

Noise Systems

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Simple Noise Systems

↕

Contours (actions)

$$C\colon \mathbb{R}^n_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n_{\geq 0}$$

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Theorem. d is a metric on T

Metrics on T compact tame $\mathbb{R}^n_{\geq 0}$ vector spaces

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Aim: calculate the stabilization of $\operatorname{rank} \colon T \to \mathbb{N}$

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Contours are called superlinear families by Bubenik, de Silva, Scott

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Aim: calculate the stabilization of $\mathrm{rank}: T \to \mathbb{N}$

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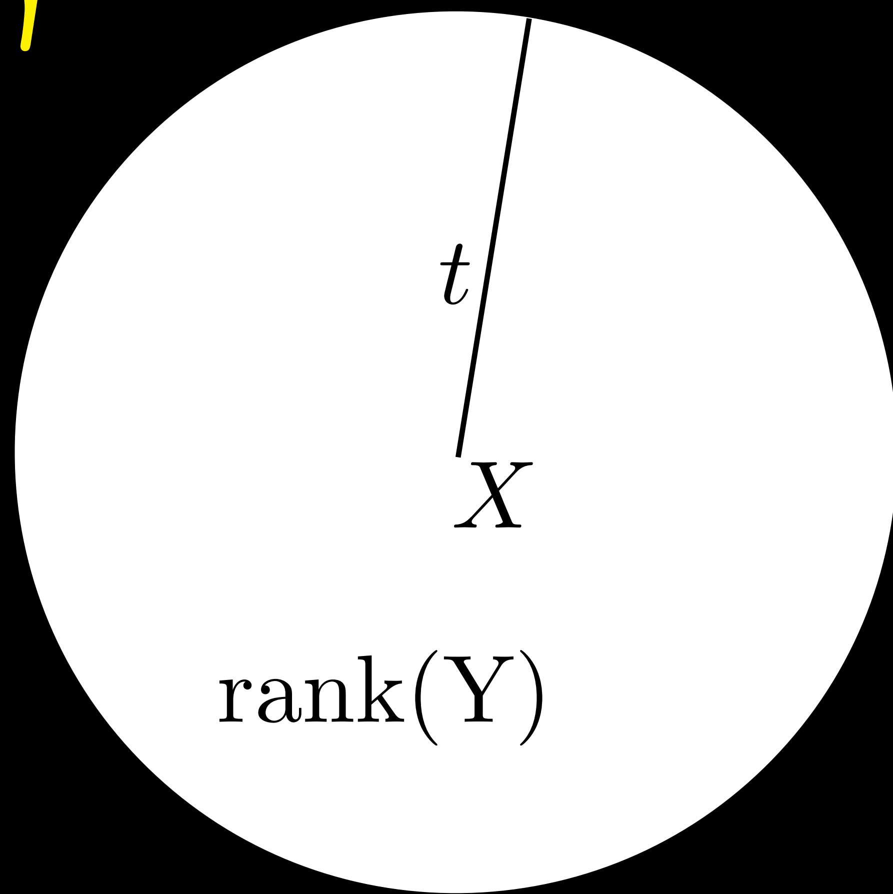

Minimize the rank
in a disc of radius t

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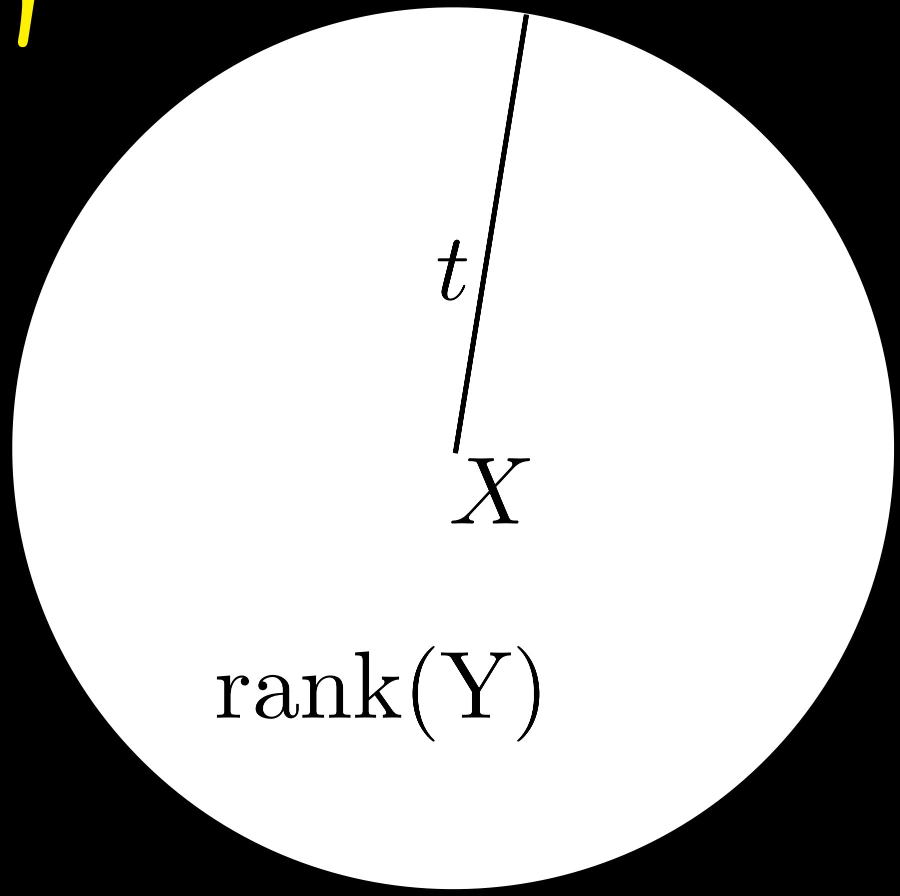
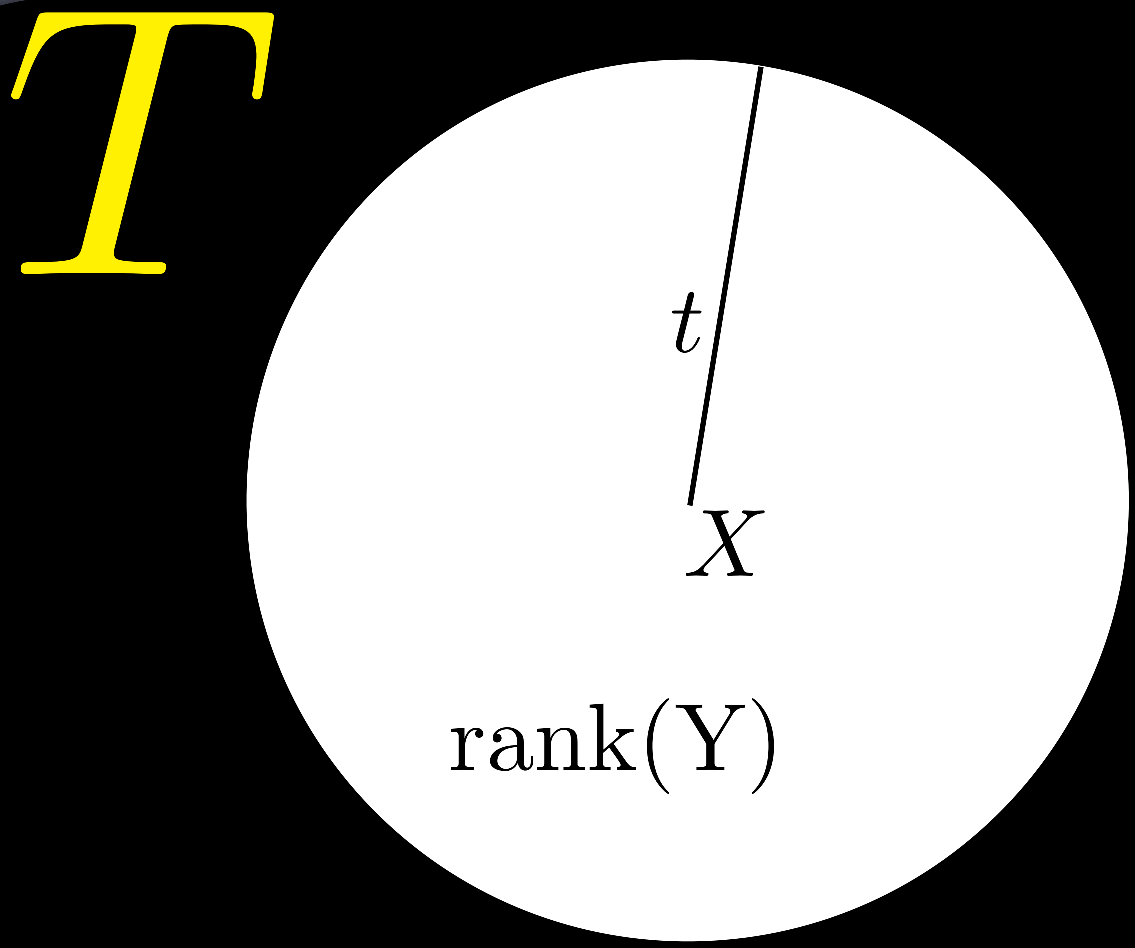


Minimize the rank in a disc of radius t

Even if the field is finite such a disc is **infinite**

Aim: calculate the stabilization of $\mathrm{rank}\colon T \to \mathbb{N}$



Minimize the rank in a disc of radius t

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Theorem

For a simple noise system, if the field is finite,

determining $\widehat{\text{rank}}(X)$

requires only finitely many calculations.

Aim: calculate the stabilization of $\text{rank}\colon T \to \mathbb{N}$

Let X, Y be compact tame $\mathbb{R}^n$ parameterized vect sp

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n=1 parameter	n>1 parameter

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X Y are isomorphic iff

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Question

Construct non isomorphic

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s.t. for any noise system

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Calculating $\widehat{\text{rank}}$

is as hard as calcutaing

the usual barcode

n>1 parameter

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Let X, Y be compact tame $\mathbb{R}^n$ parameterized vect sp

$$\widehat{\text{rank}}(X)(t)$$

$$||$$

$$|\{\text{bars } [b,d) \text{ in } X \mid C(b,t) < d\}|$$

ameter

n isomorphic

Y

noise system

$= \widehat{\text{rank}}(Y)$

Calculating $\widehat{\text{rank}}$
is as hard as calcutaing
the usual barcode

Let X, Y be compact tame $\mathbb{R}^n$ parameterized vect sp

n=1 parameter

Theorem

X Y are isomorphic iff

for any contour

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Construct non isomorphic

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O. Gäfvert theorem:

Calculating $\widehat{\text{rank}}$

is an NP hard problem

n=1 Implementing contours

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A density is an ingerable function $f : \mathbb{R}_{\geq} \to \mathbb{R}_{>0}$

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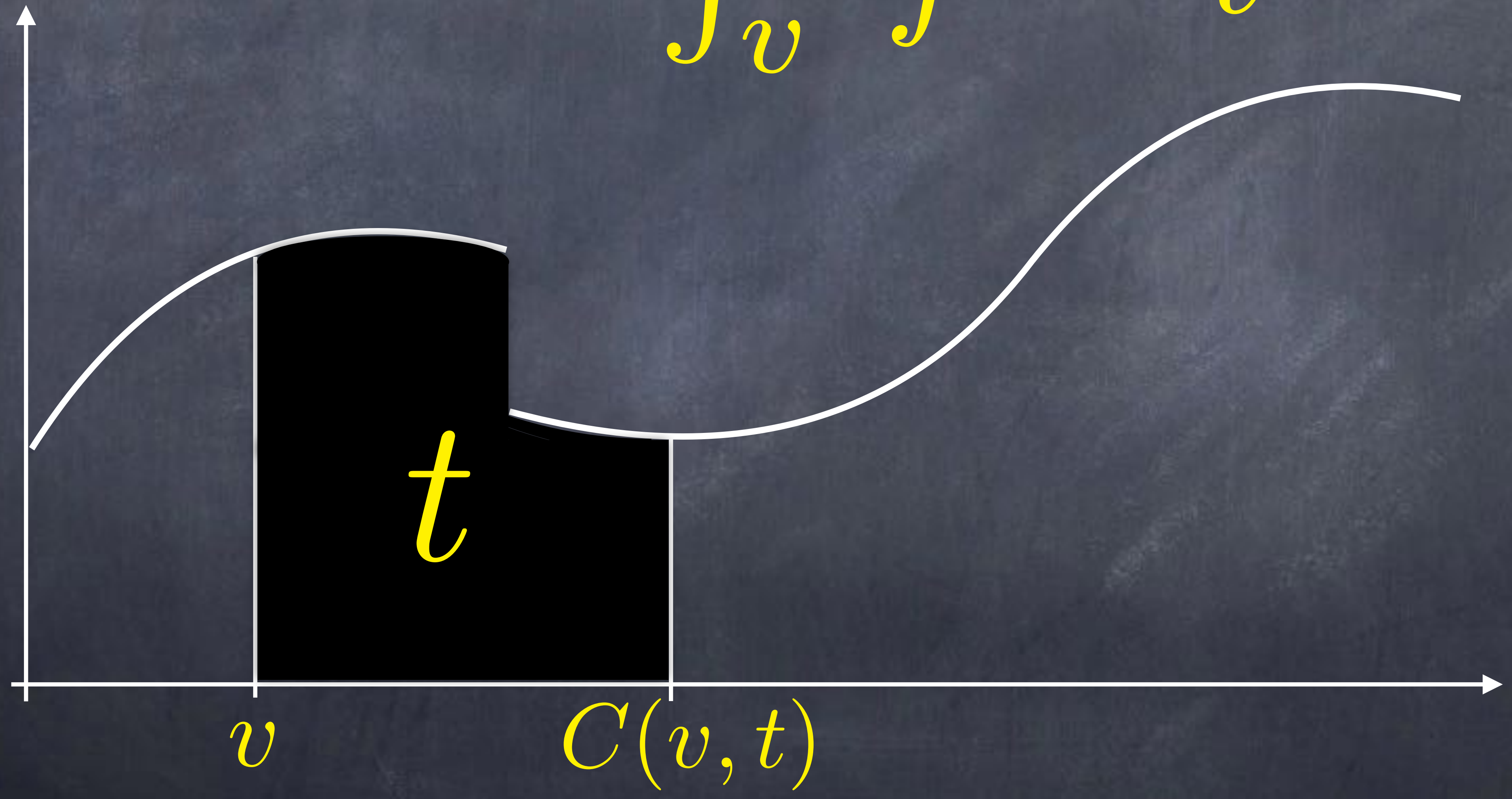

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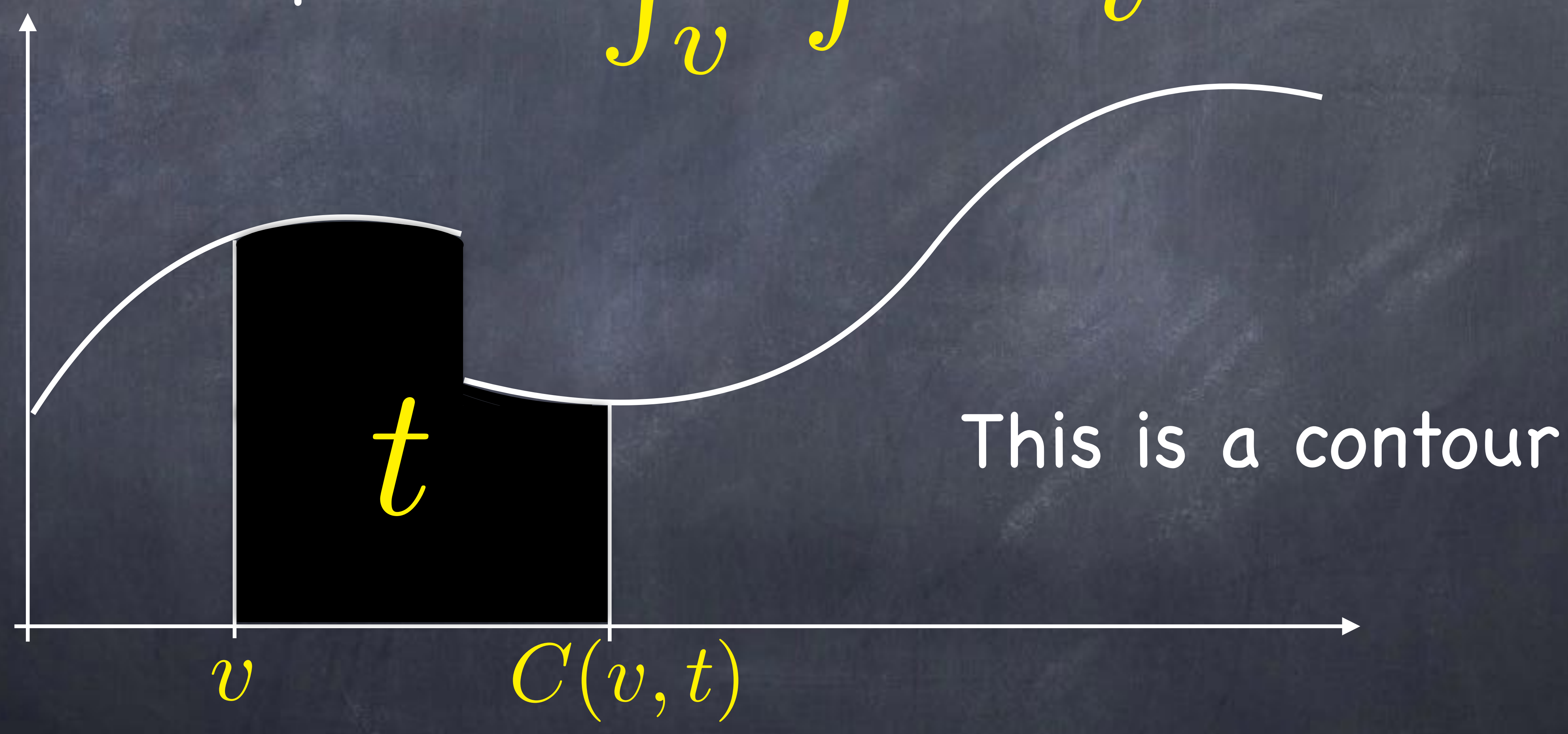
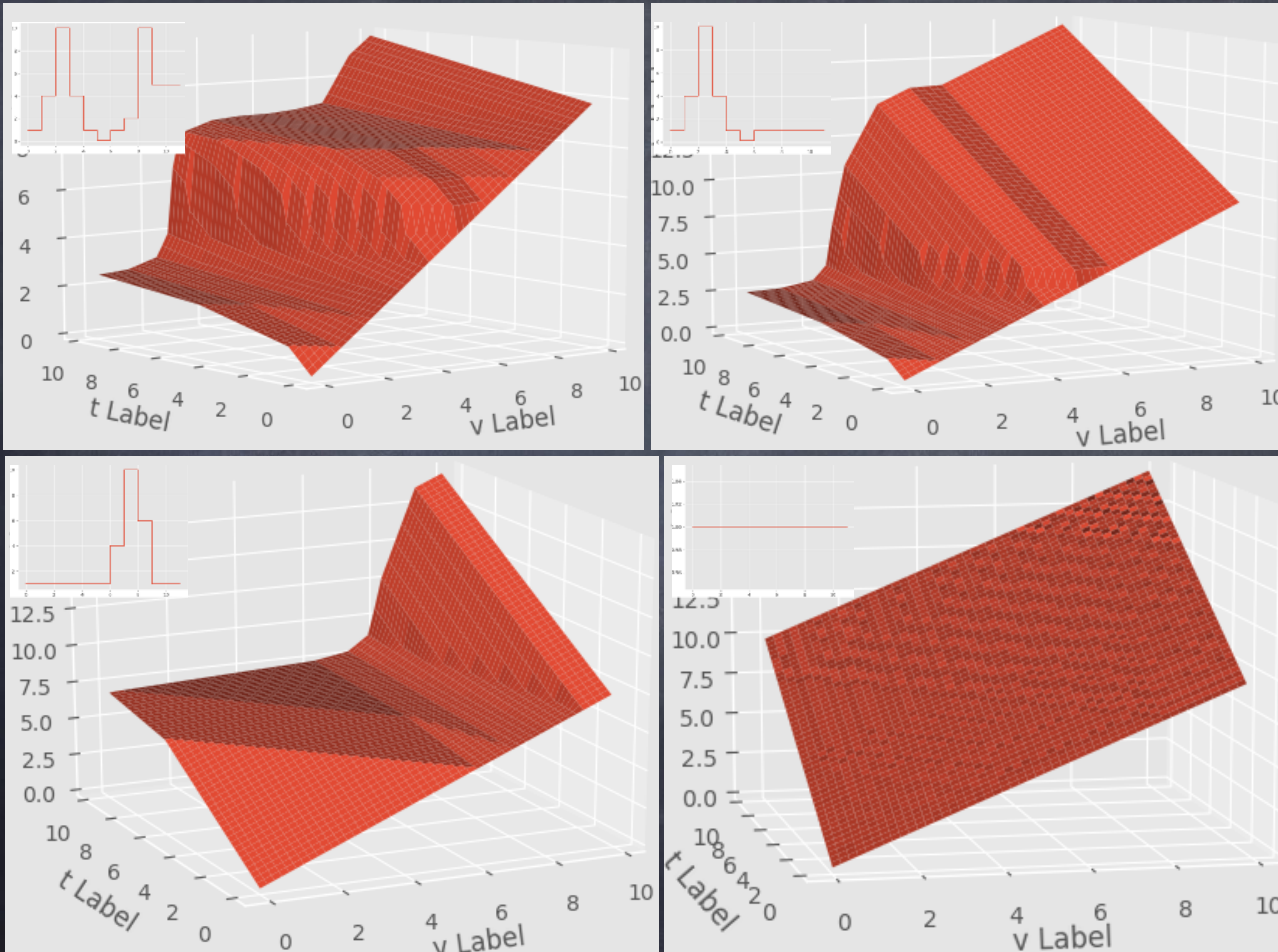
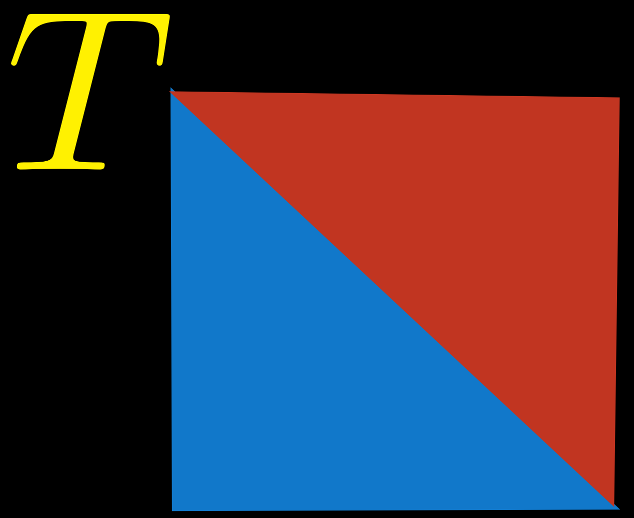




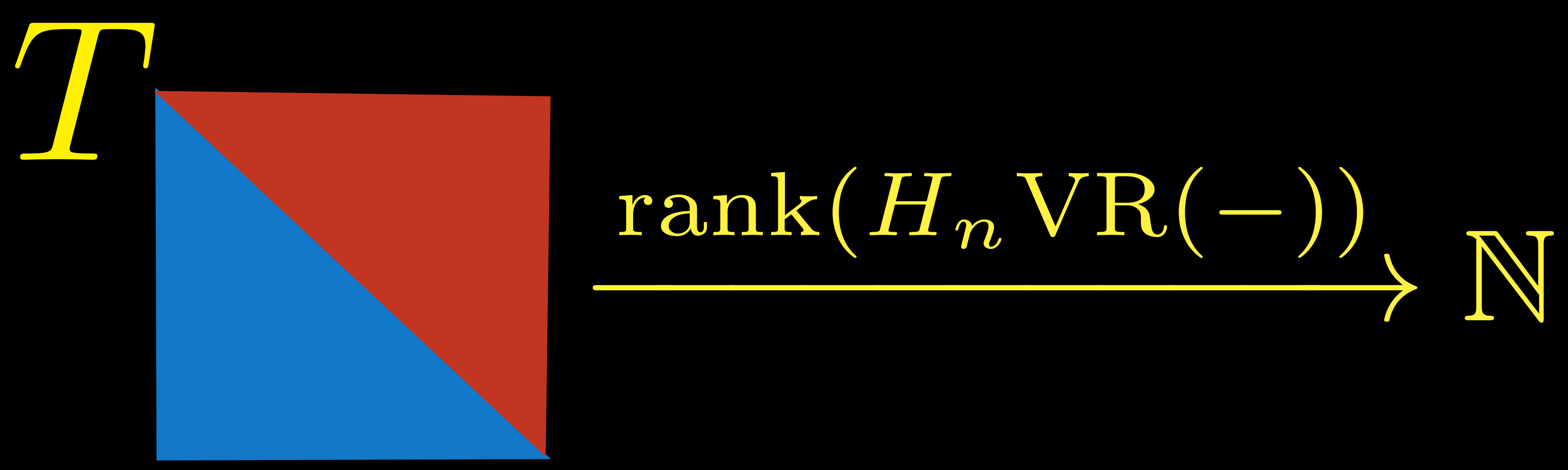
t Label
v Label
t Label
v Label
t Label
v Label
t Label
v Label

Contour learning

Contour learning



Contour learning



Contour learning

Density functions

$$f : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$$

T

$$\xrightarrow{\operatorname{rank}(H_n \mathrm{VR}(-))} \mathbb{N}$$

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Contour learning

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$f : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$ $\xrightarrow{\psi}$ $\mathbb{R}$

$$\psi(f) = \inf\{ \int |\widehat{\mathrm{rank}}(X) - \widehat{\mathrm{rank}}(Y) \mid X, Y \}$$

T

$$\xrightarrow{\widehat{\mathrm{rank}}(H_n \mathrm{VR}(-))} \mathrm{Funk}(\mathbb{R}_{\geq 0}, \mathbb{R}_{\geq 0})$$

Contour learning: Maximizing ψ

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6 point processes on the unit square, containing 200 points on average

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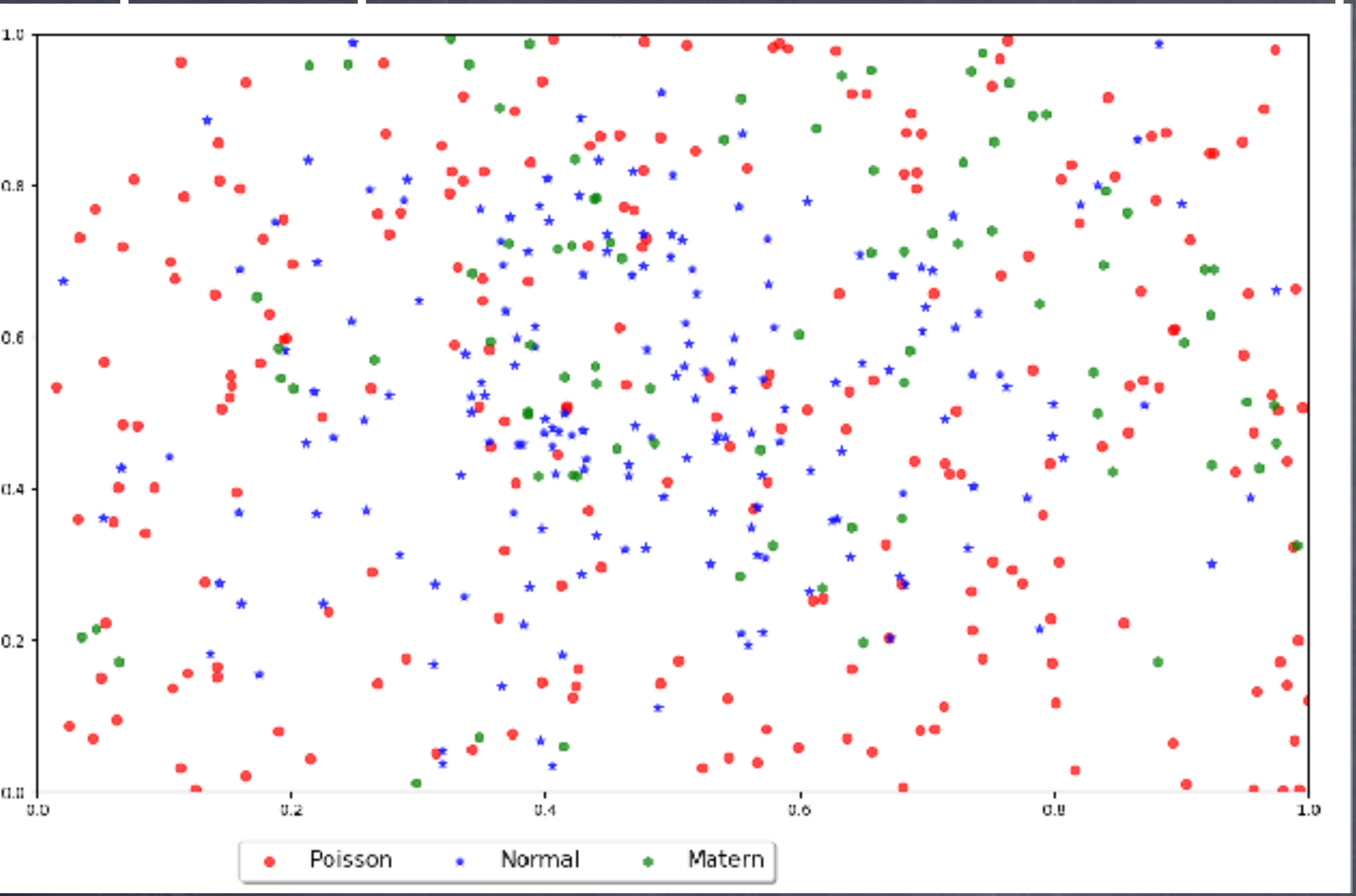

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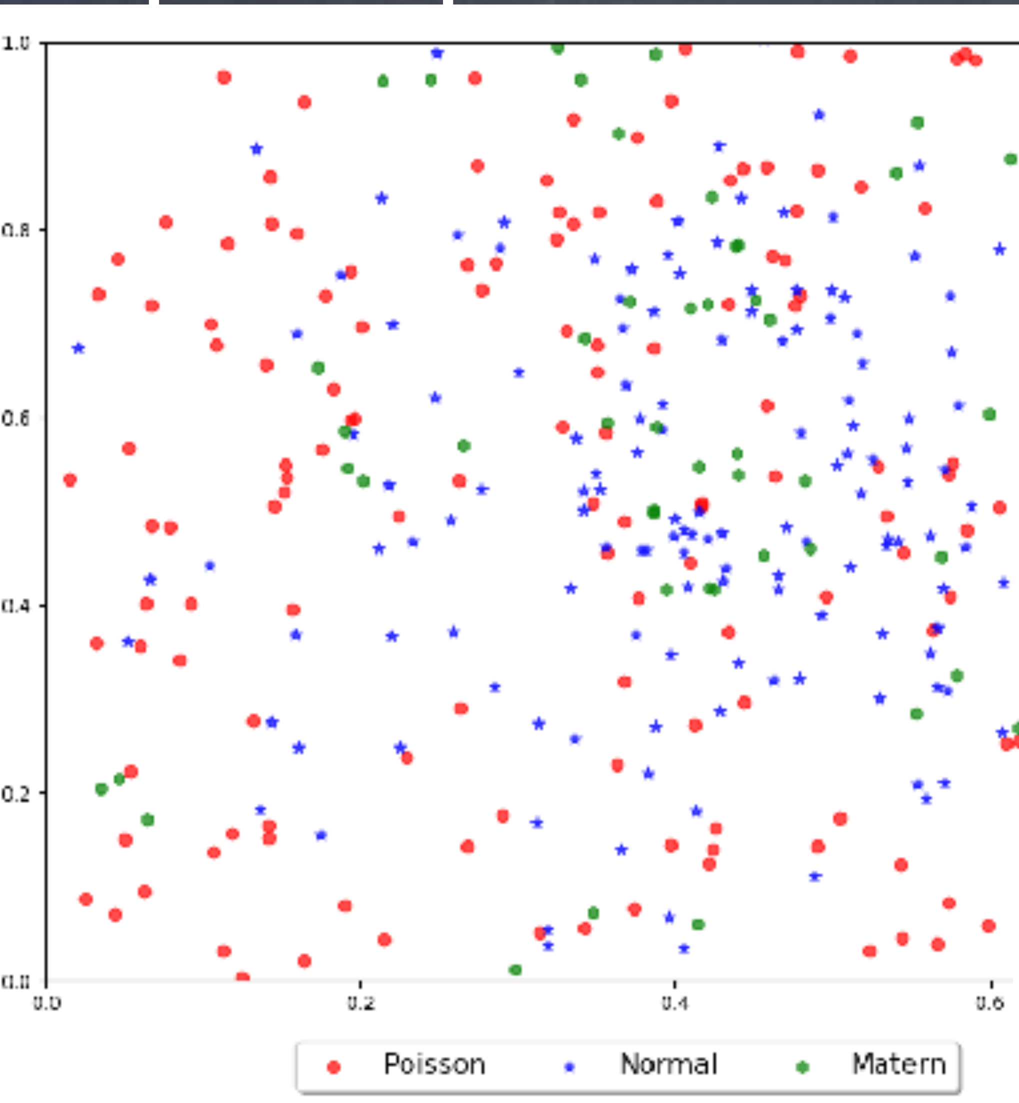

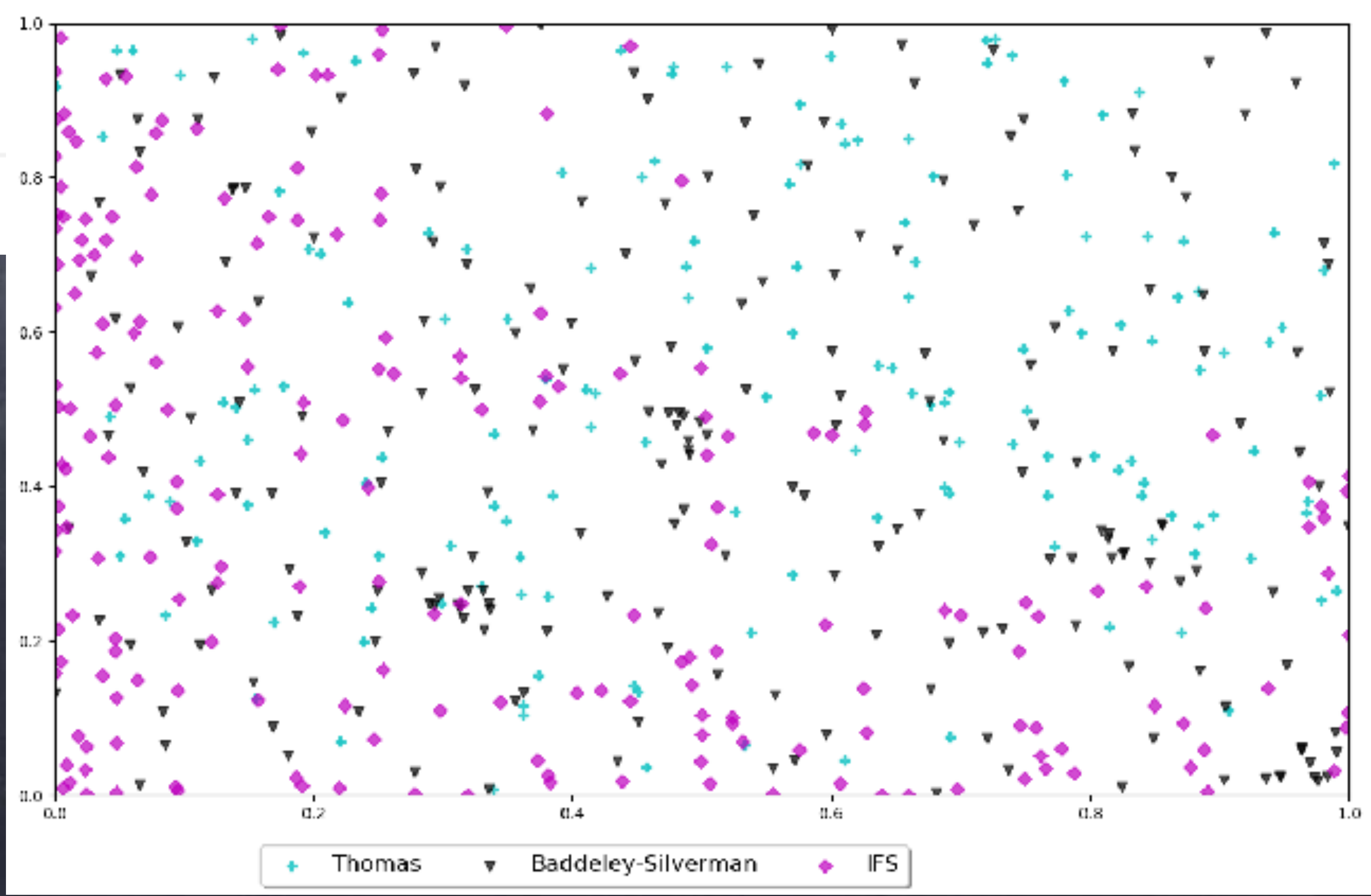

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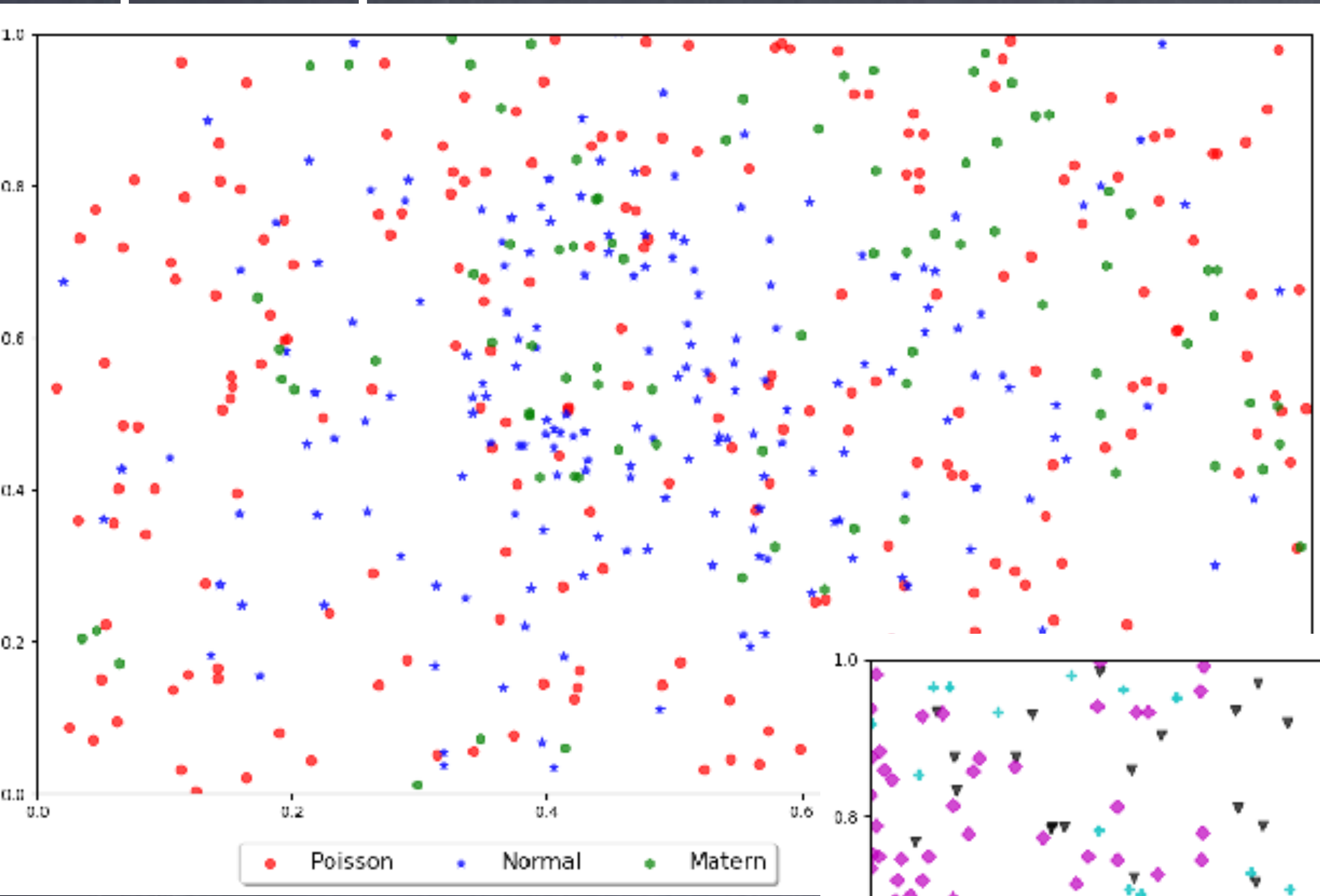
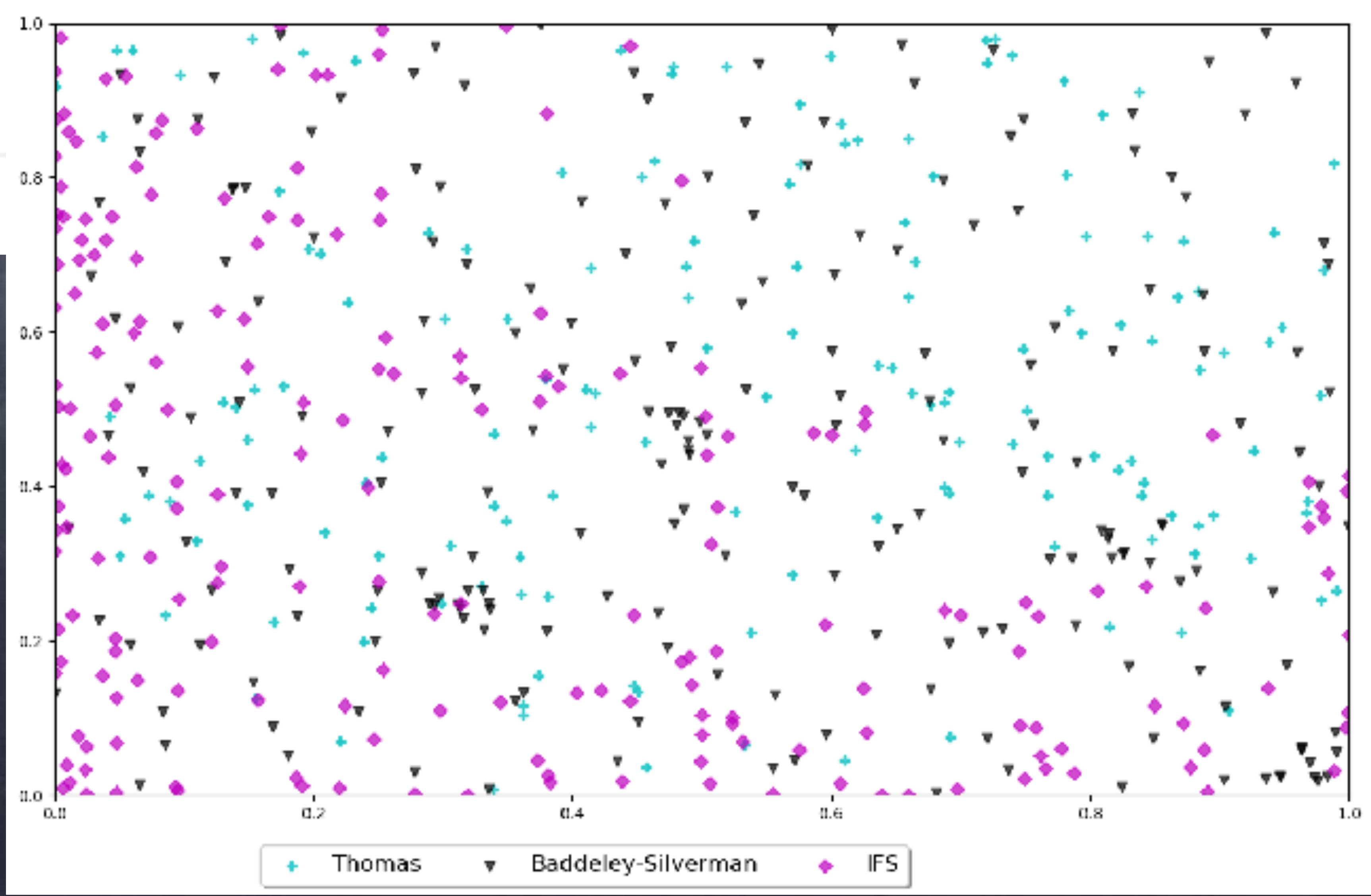


500 instances of each process was generated



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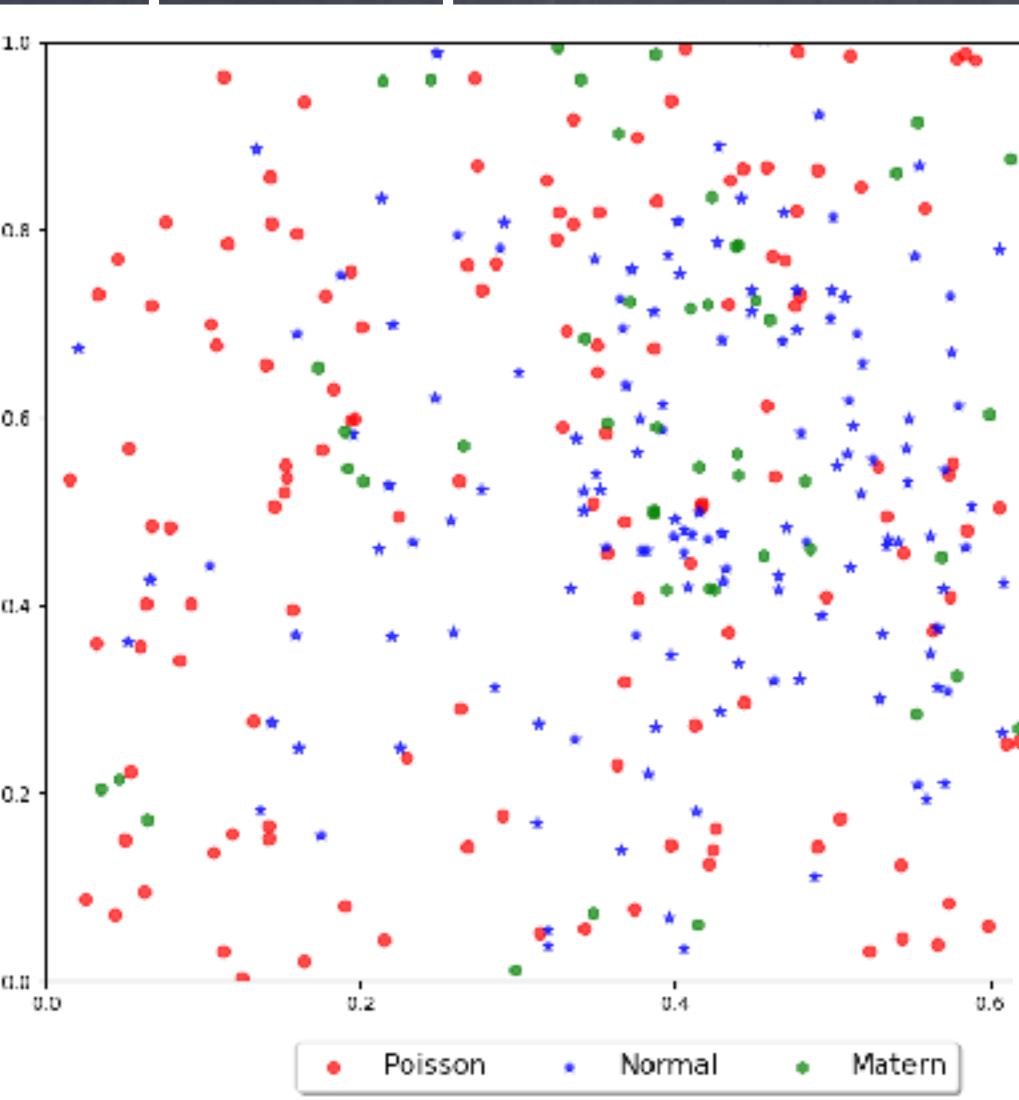

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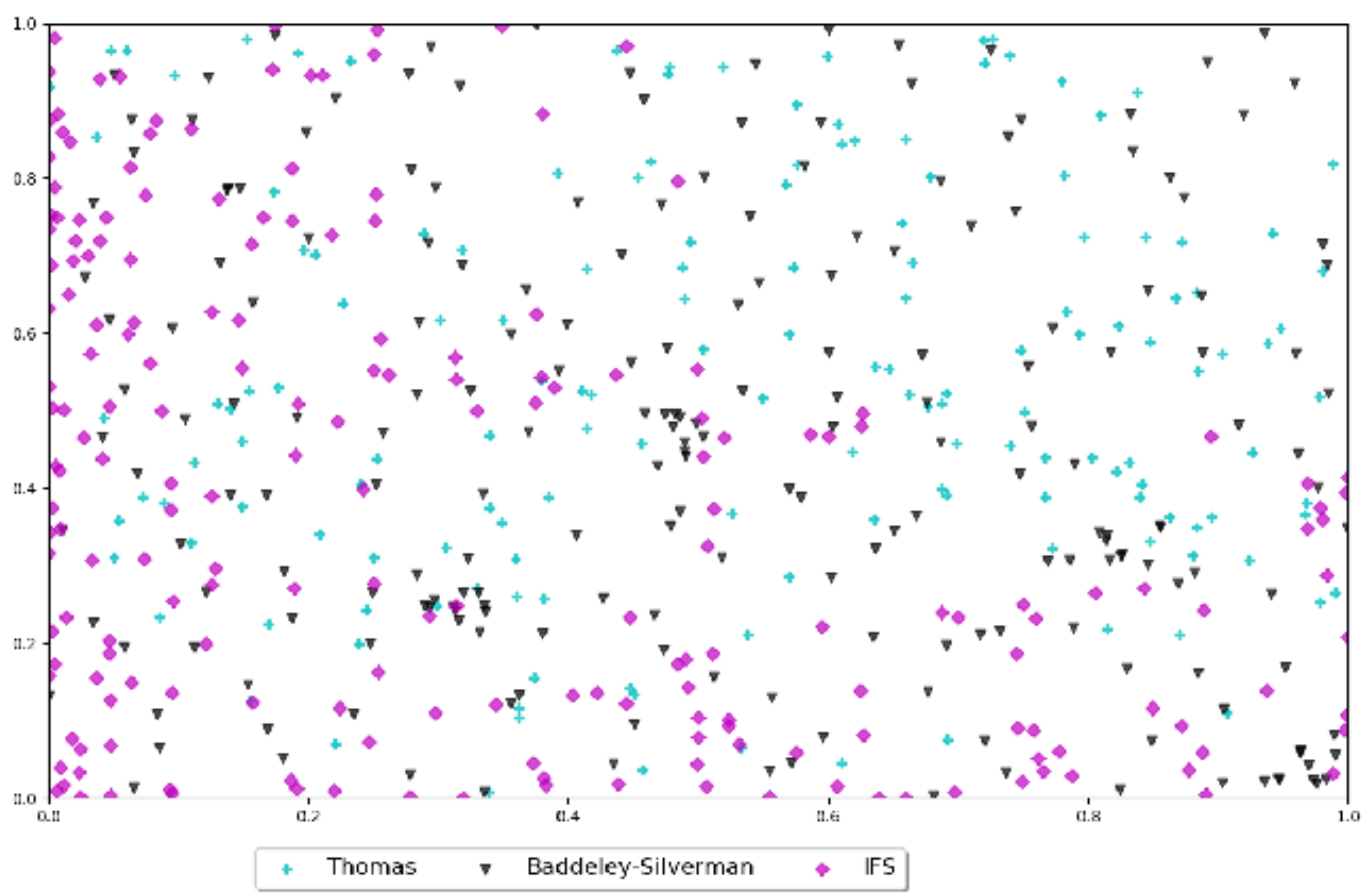
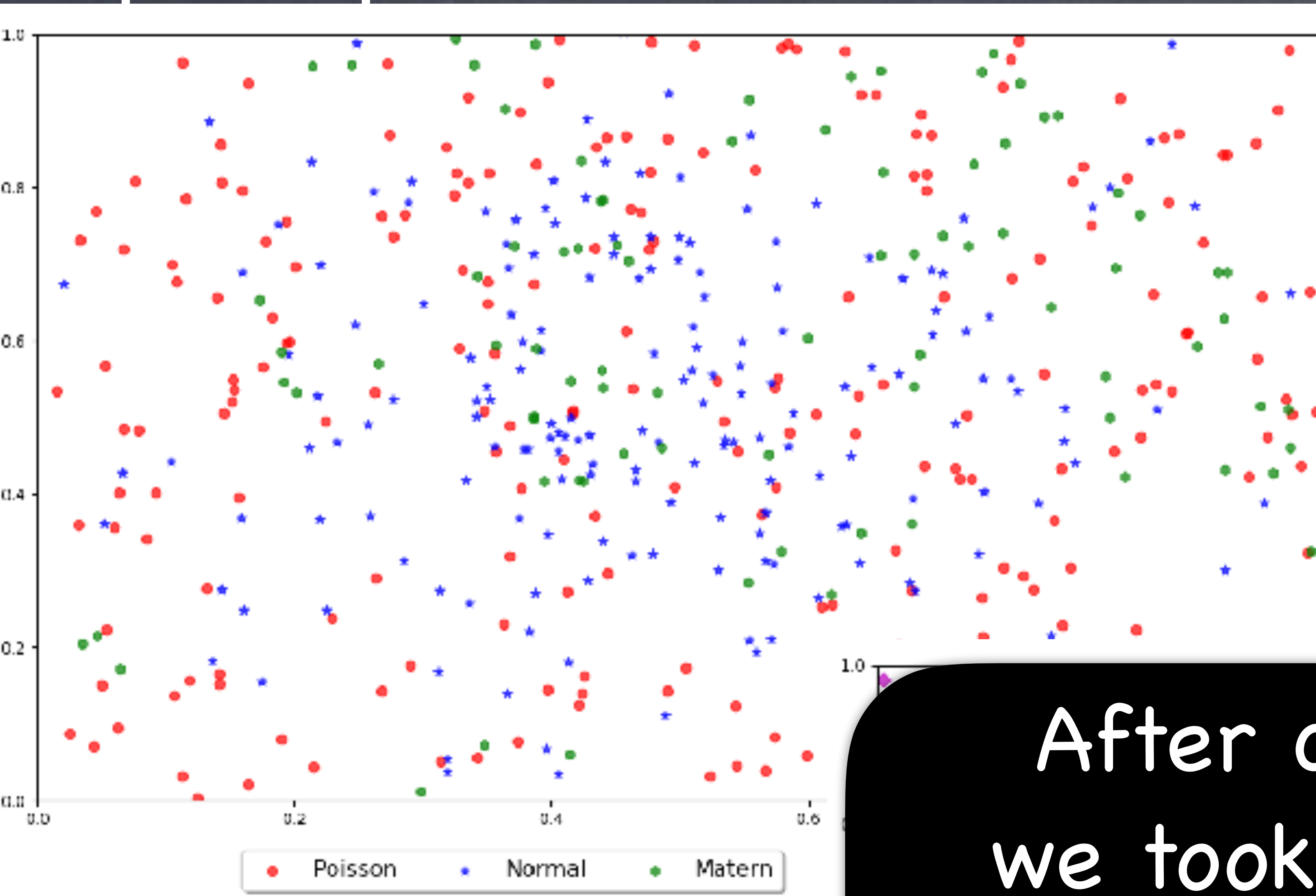


200 used for training

300 used for testing

6 point processes on the unit square, containing 200 points on average



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After choosing a contour we took averages of stable ranks of the 200 processes in the training set

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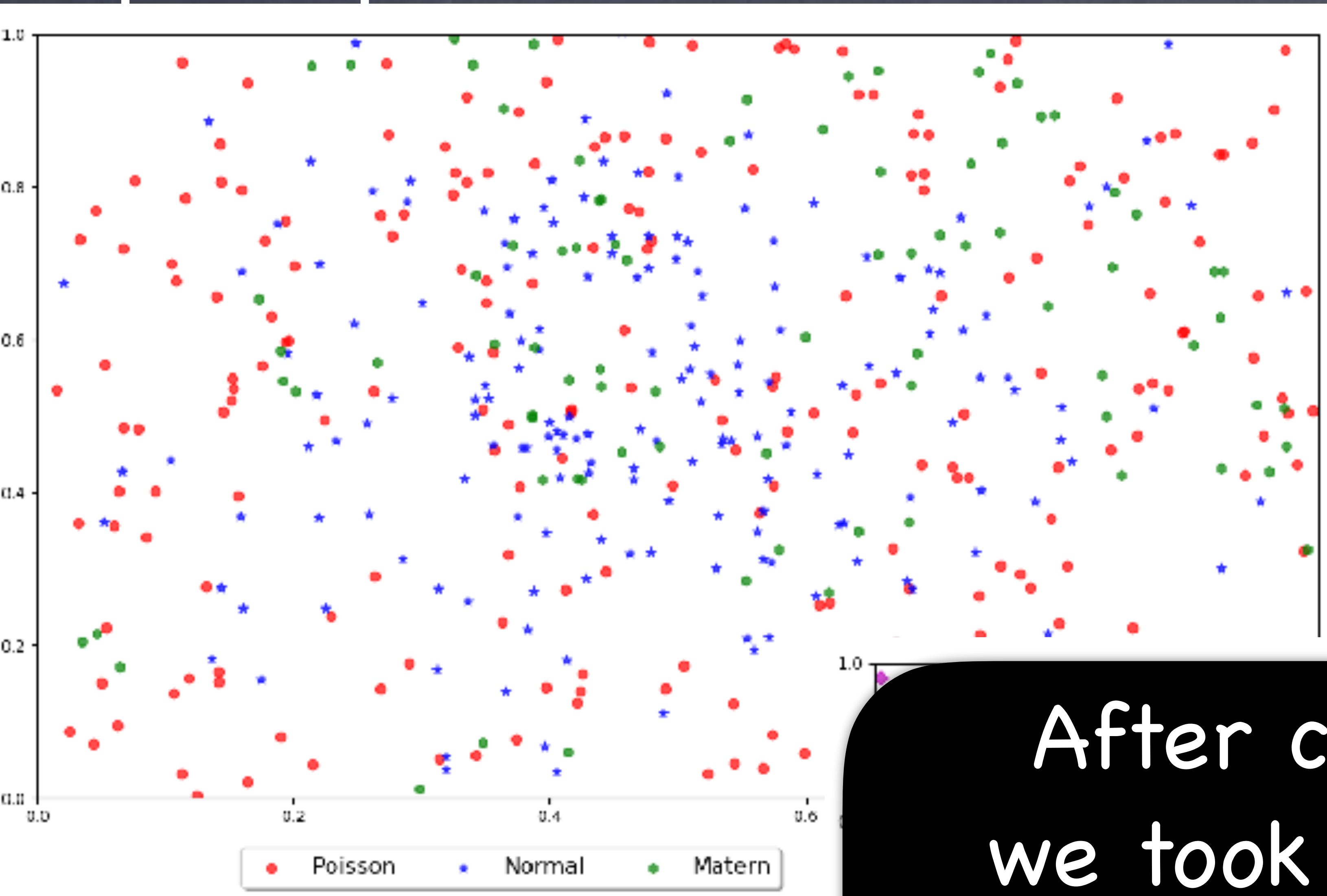

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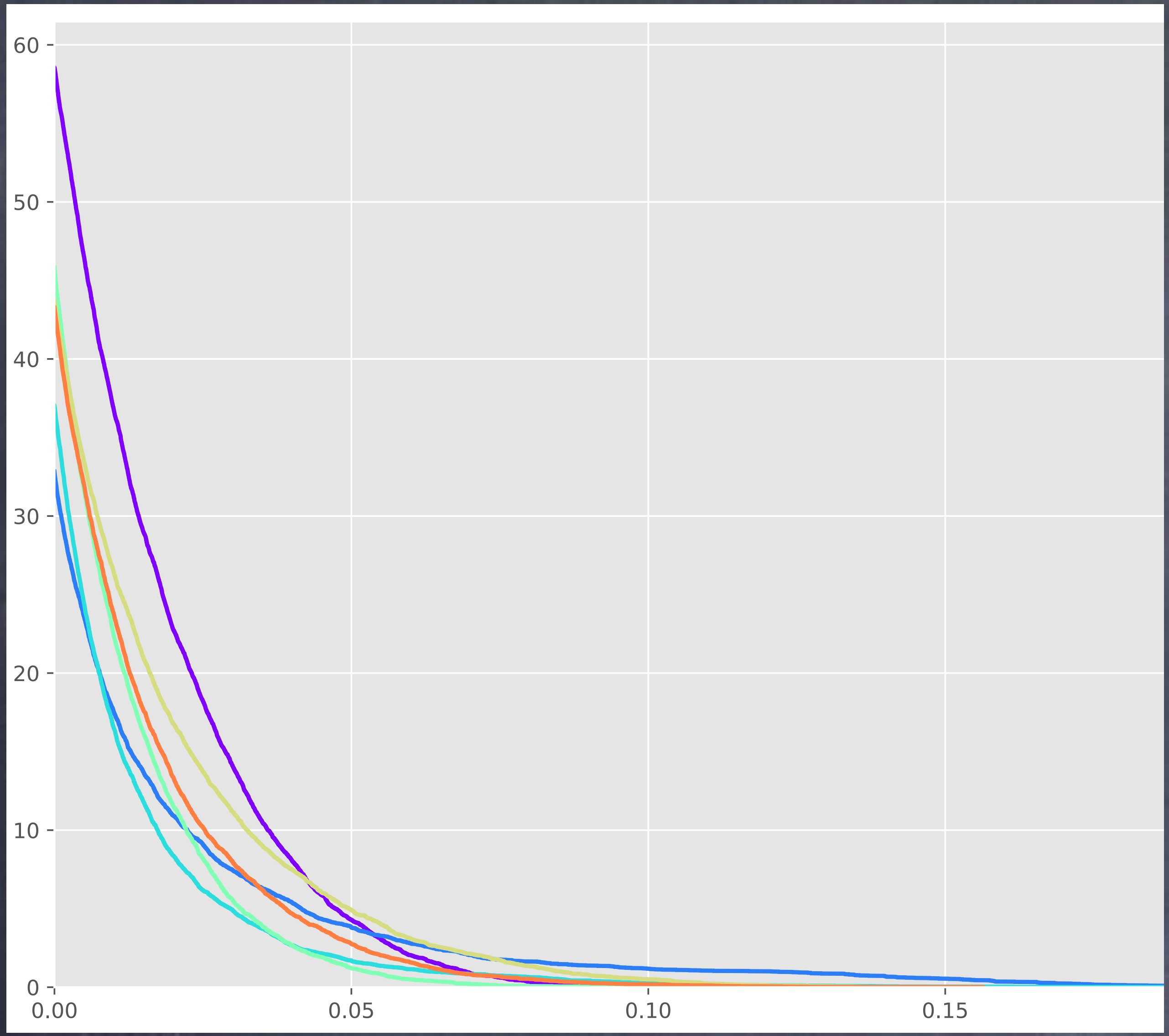

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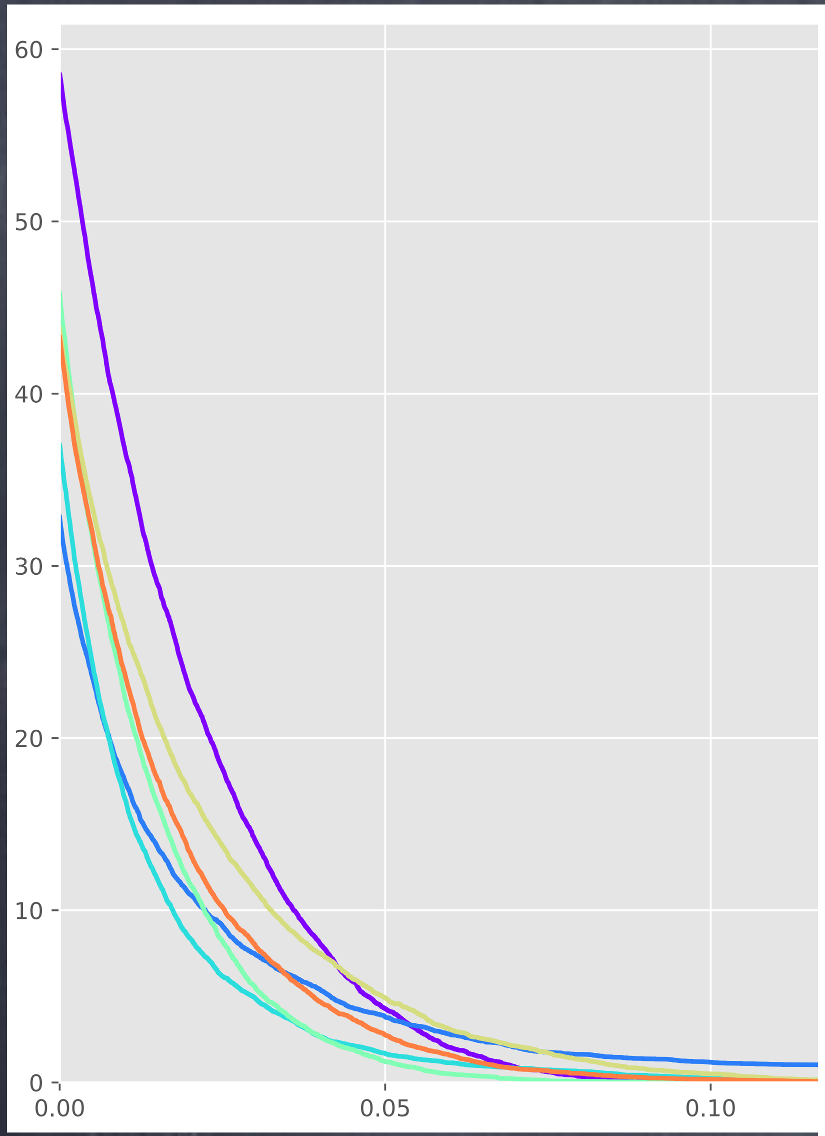

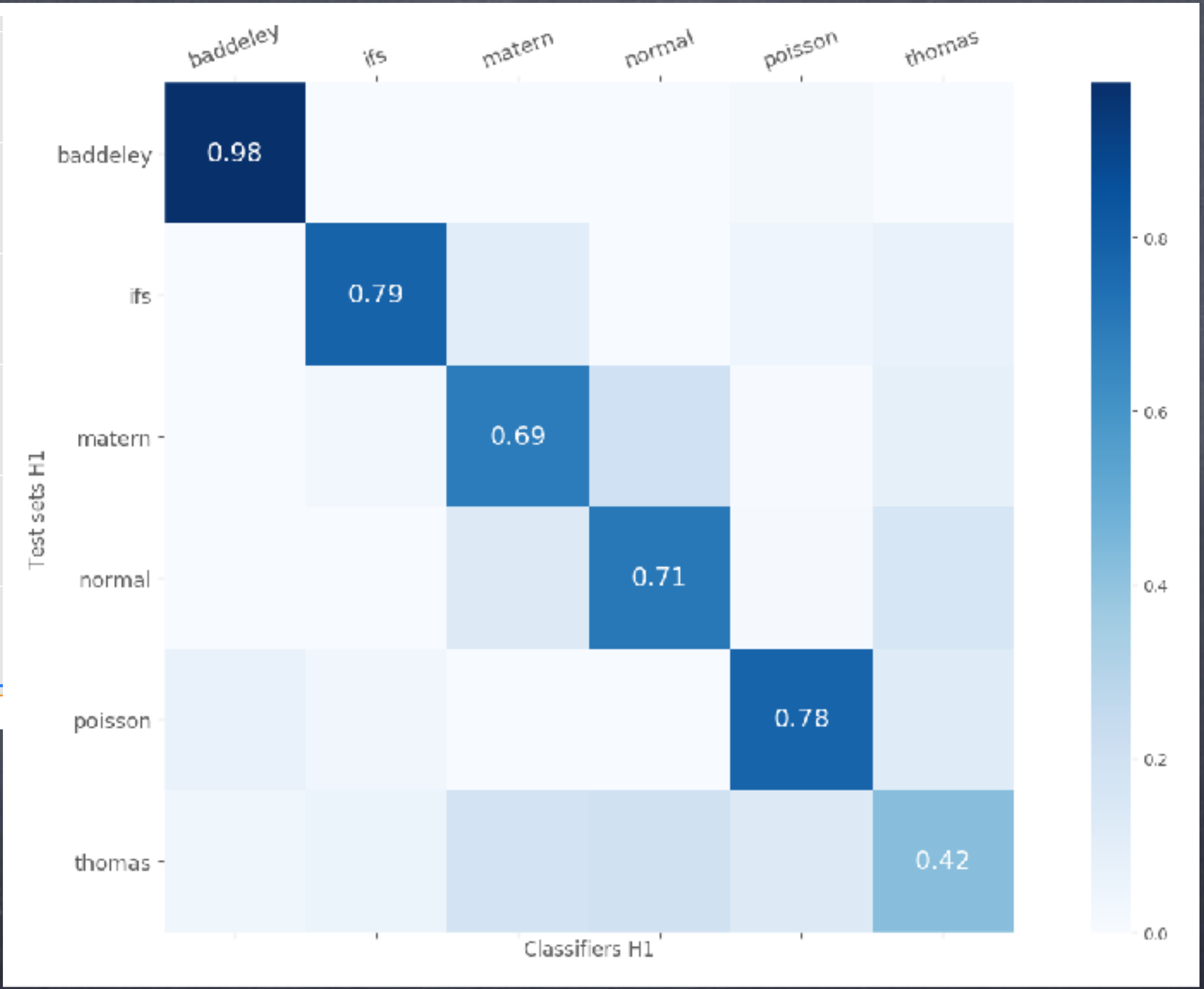

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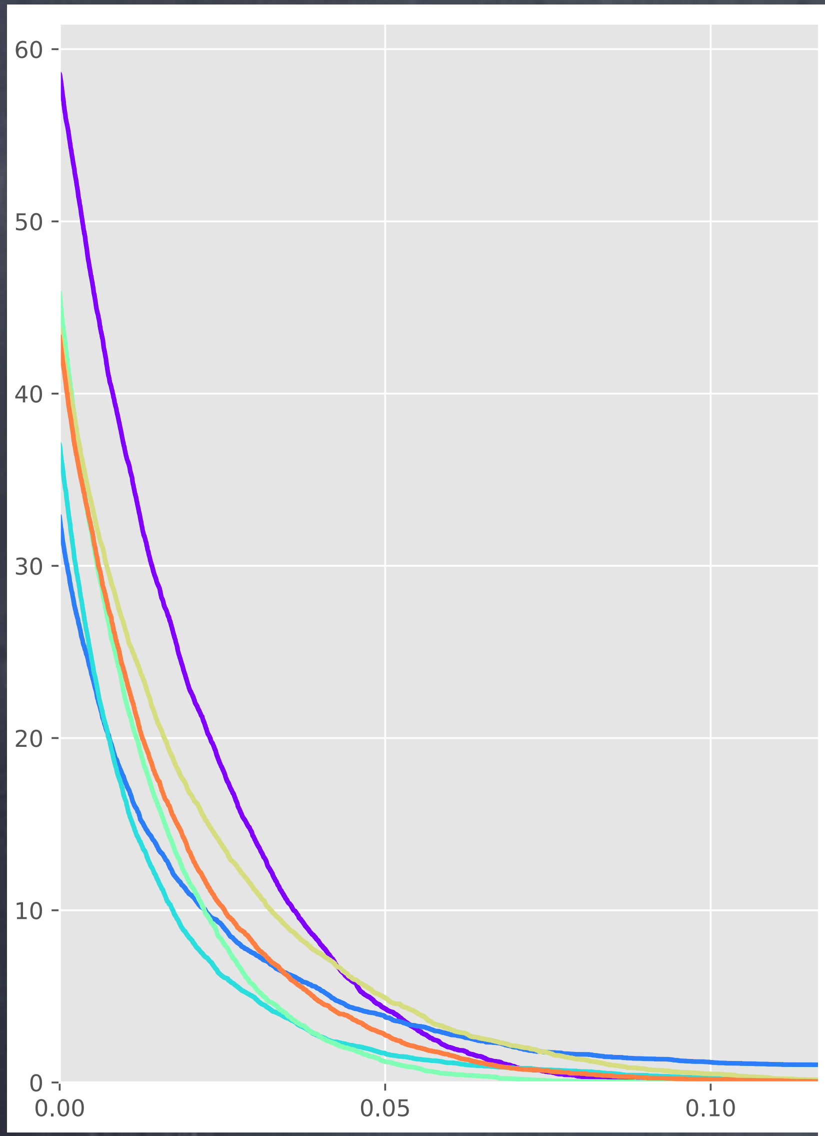
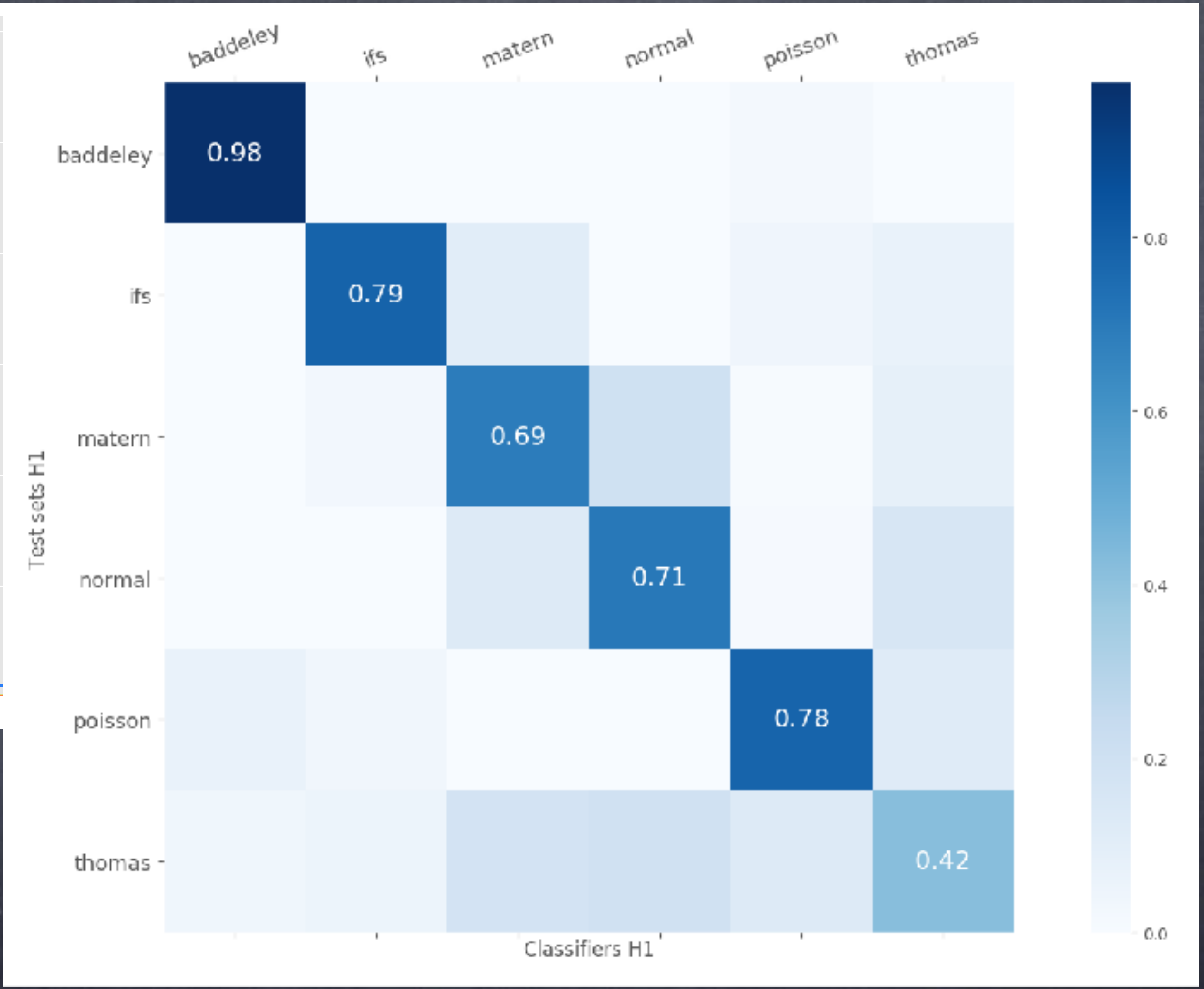
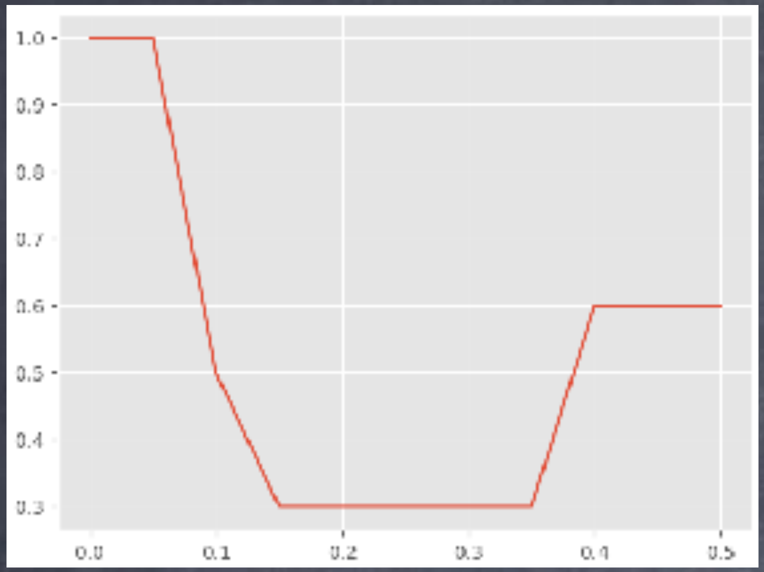


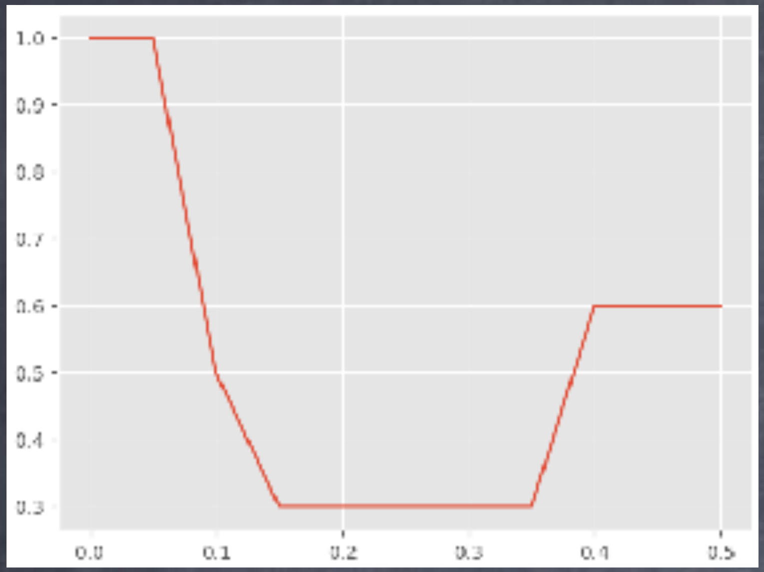


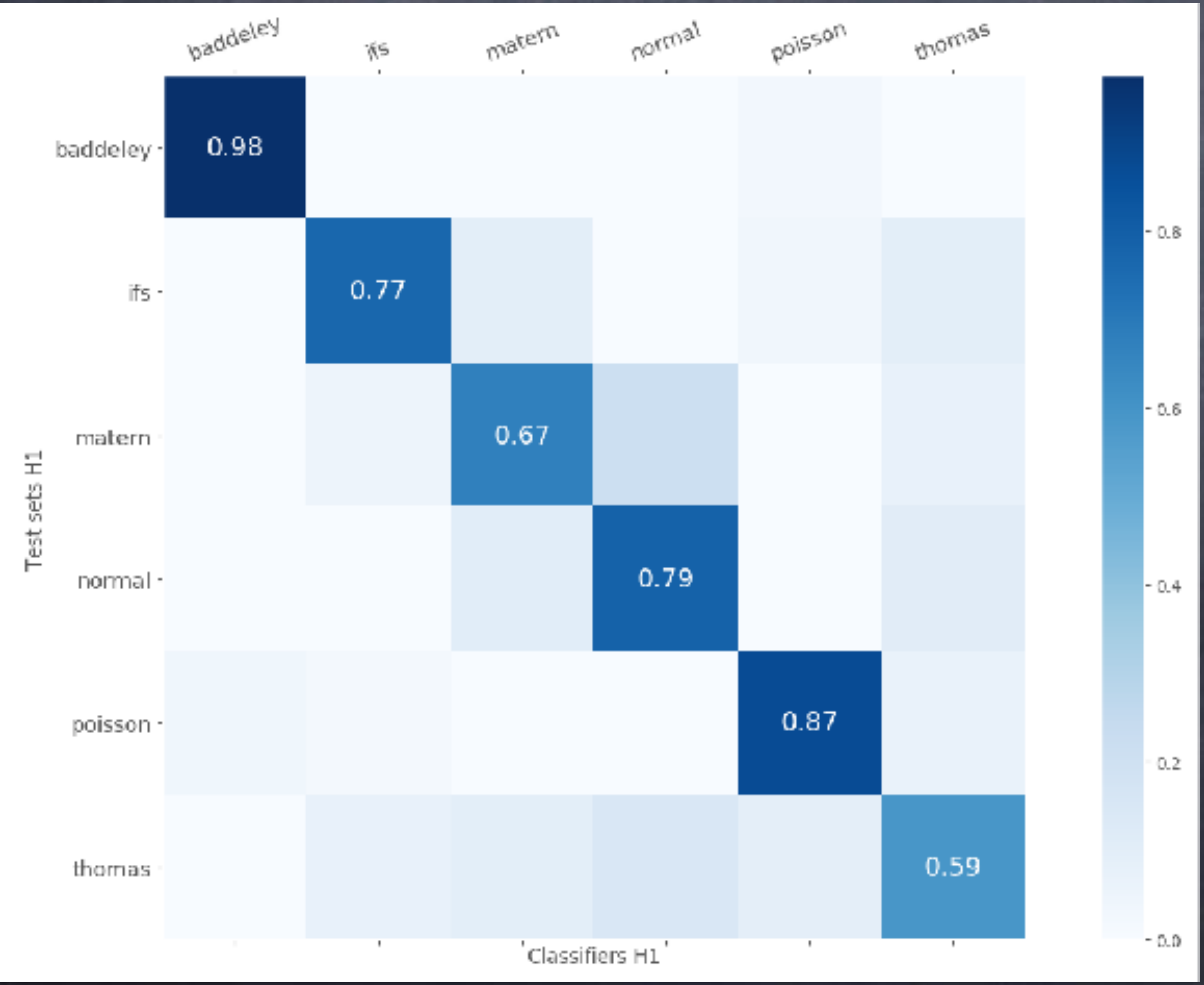
Averages after 20 X cross validation

Density:

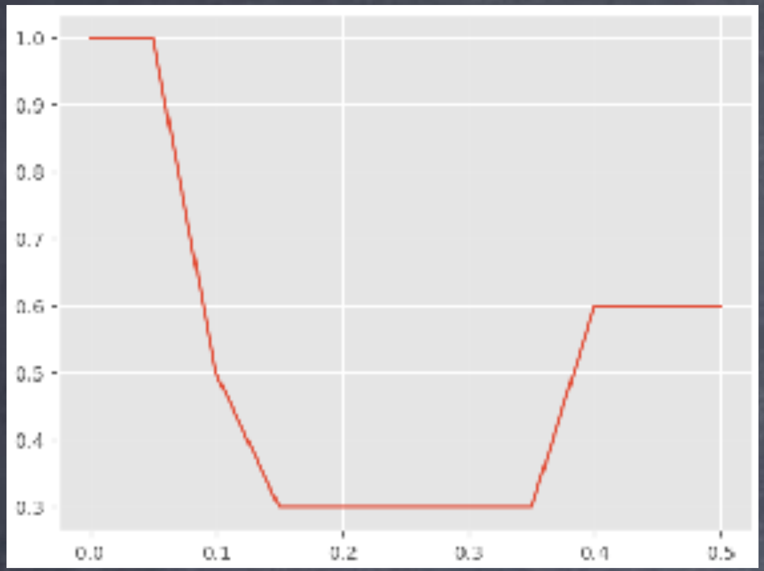


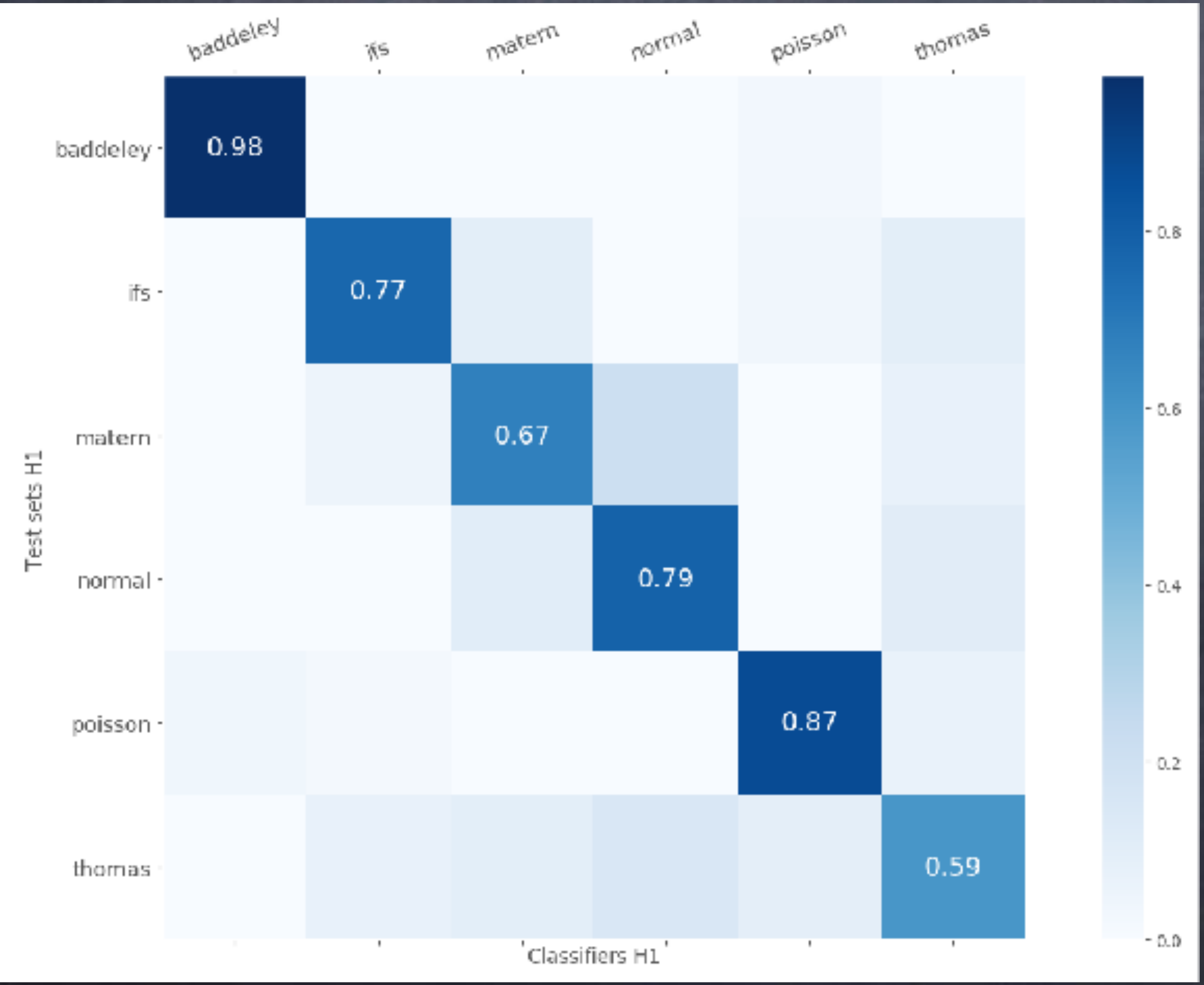
Density:


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.0
0.1
0.2
0.3
0.4
0.5


baddeley
ifs
matern
normal
poisson
thomas
0.98
0.77
0.67
0.79
0.87
0.59
Test sets H1
Classifiers H1
0.8
0.6
0.4
0.2
0.0

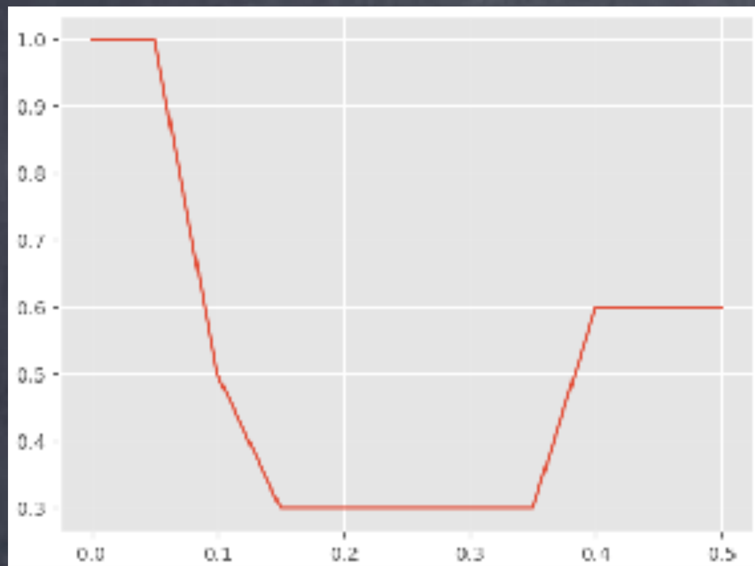
Density:



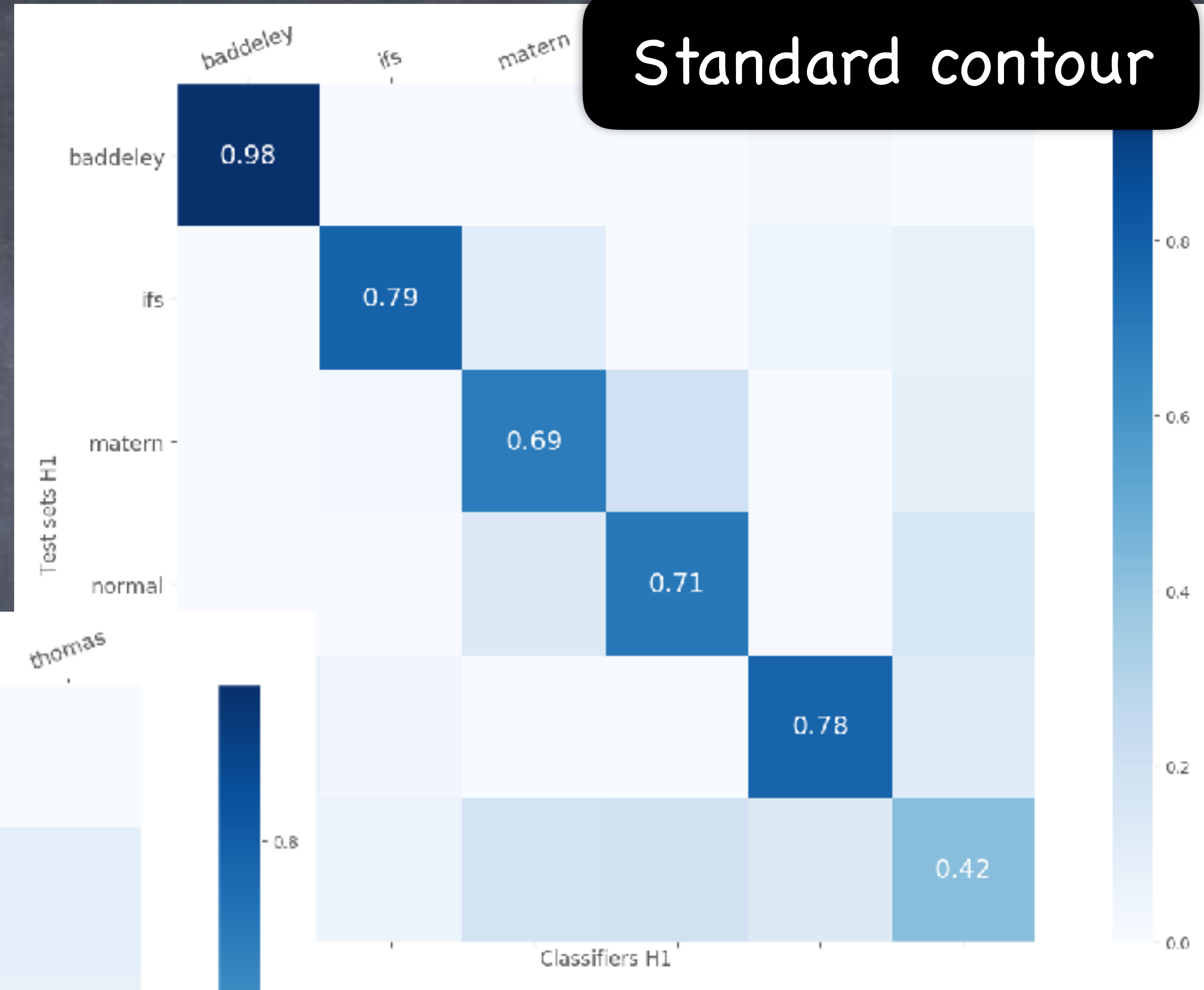


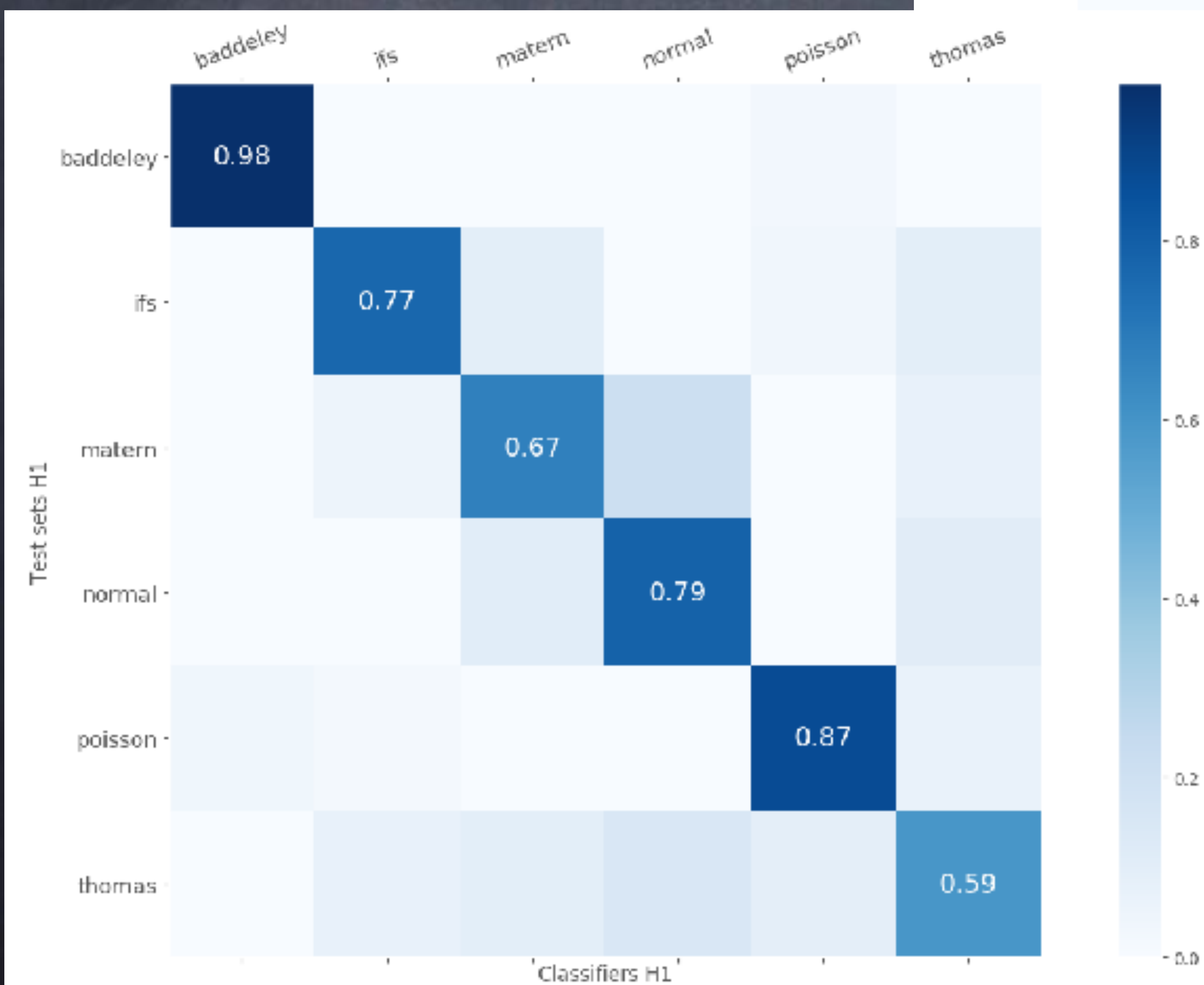
Averages
after 20 X cross
validation

Density:



Standard contour





Averages after 20 X cross validation

Henri Riihimäki, forest tree data:

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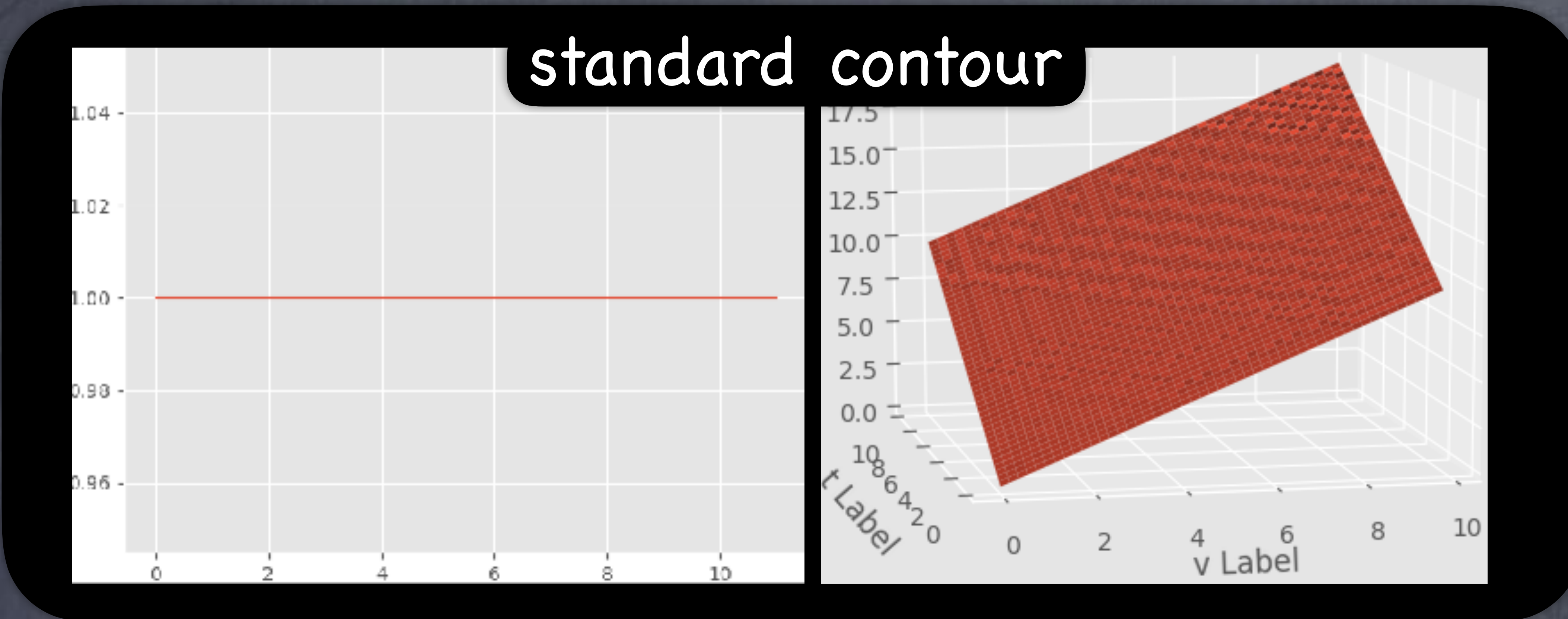

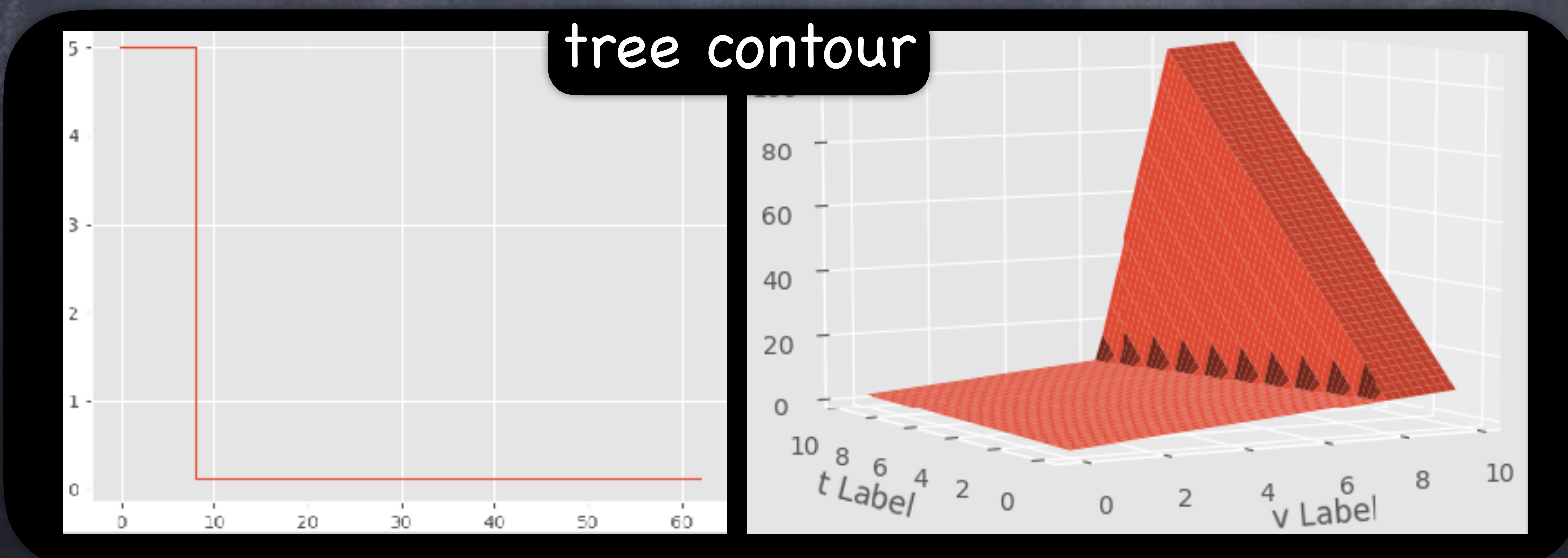

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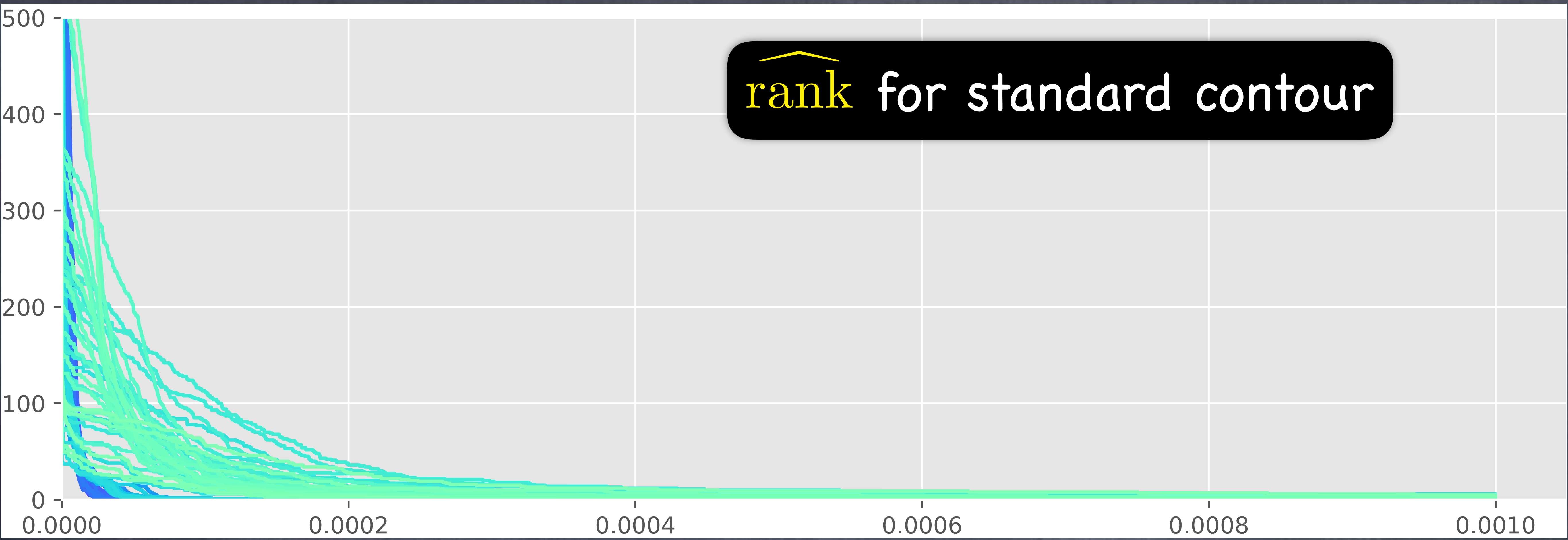

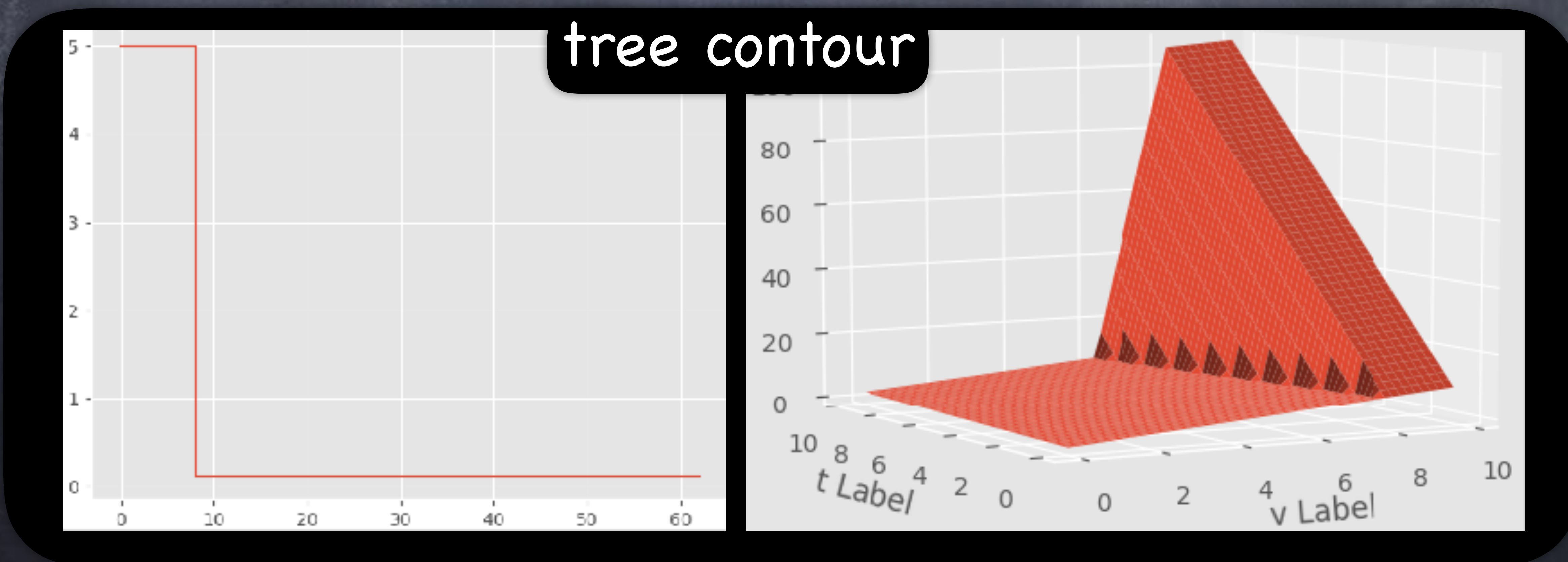

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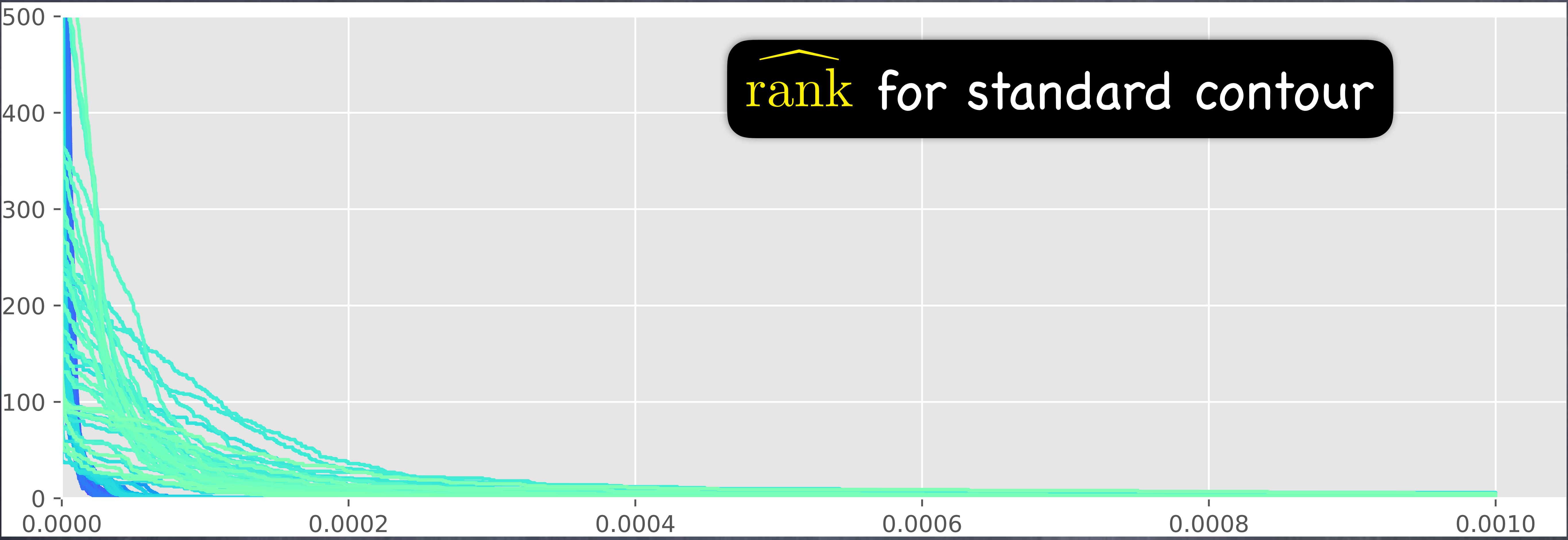
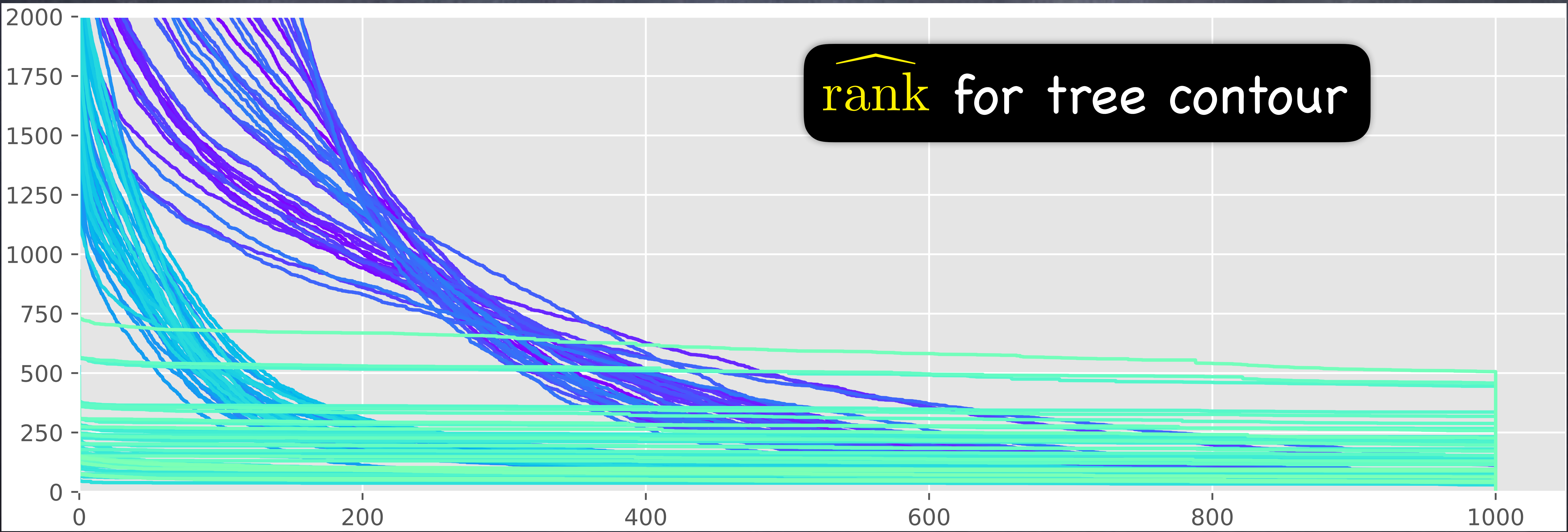
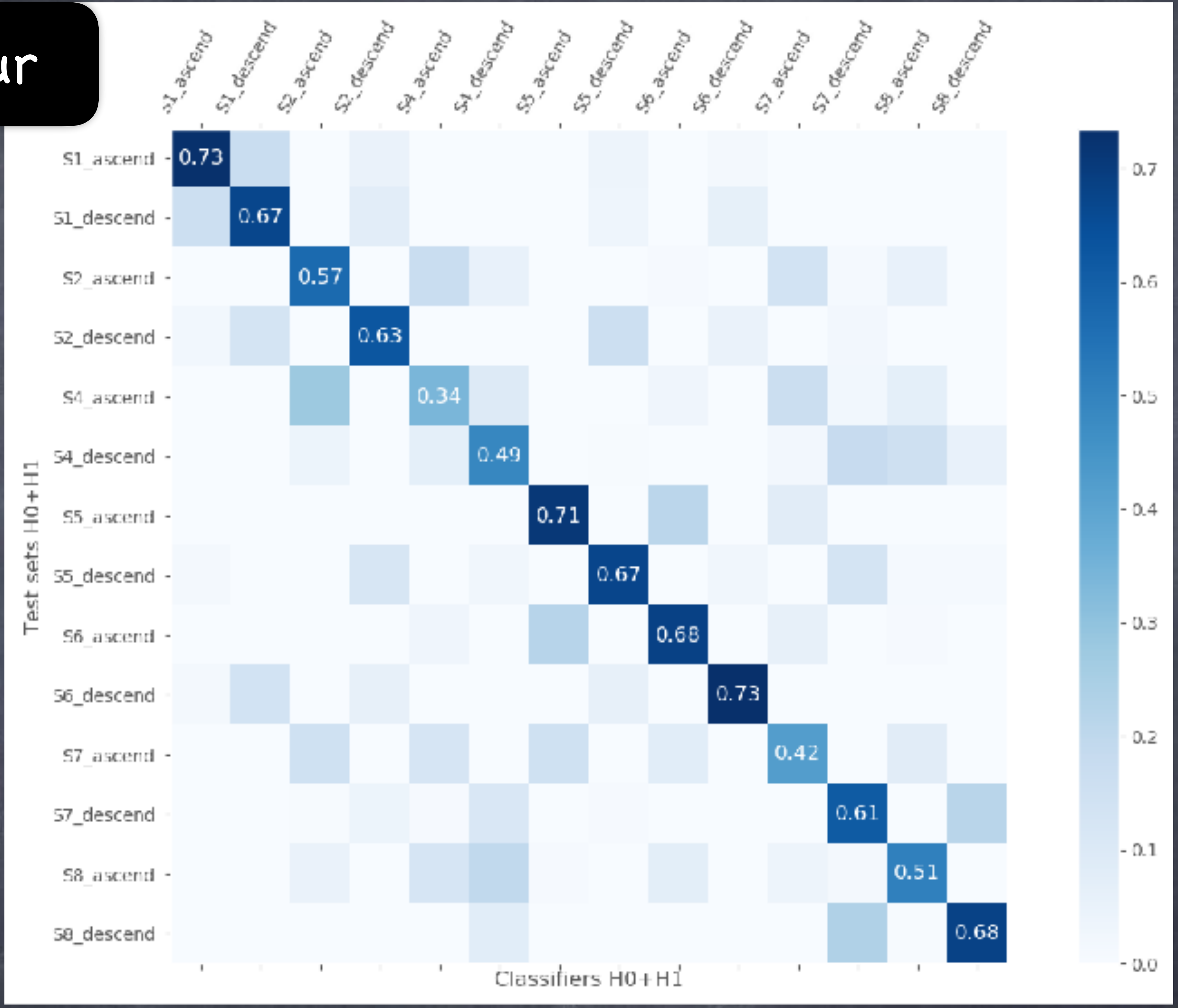




PAMAP2 DATA (www.pamap.org):

- 7 subjects were equiped with 3 sensors (IMU)
- producing 28 measurments every 0.1 second
- in the following analysis we concidered two activities of accending and decending stairs.
- for every subject we have two time series in 28 dim space
- for each series we sample 100 time slots without replacement
- we took 100 such samplings
- 40 are used as training and 60 for testing
- after choosing a contour, the average of the 40 train data are used as classifiers

Standard contour



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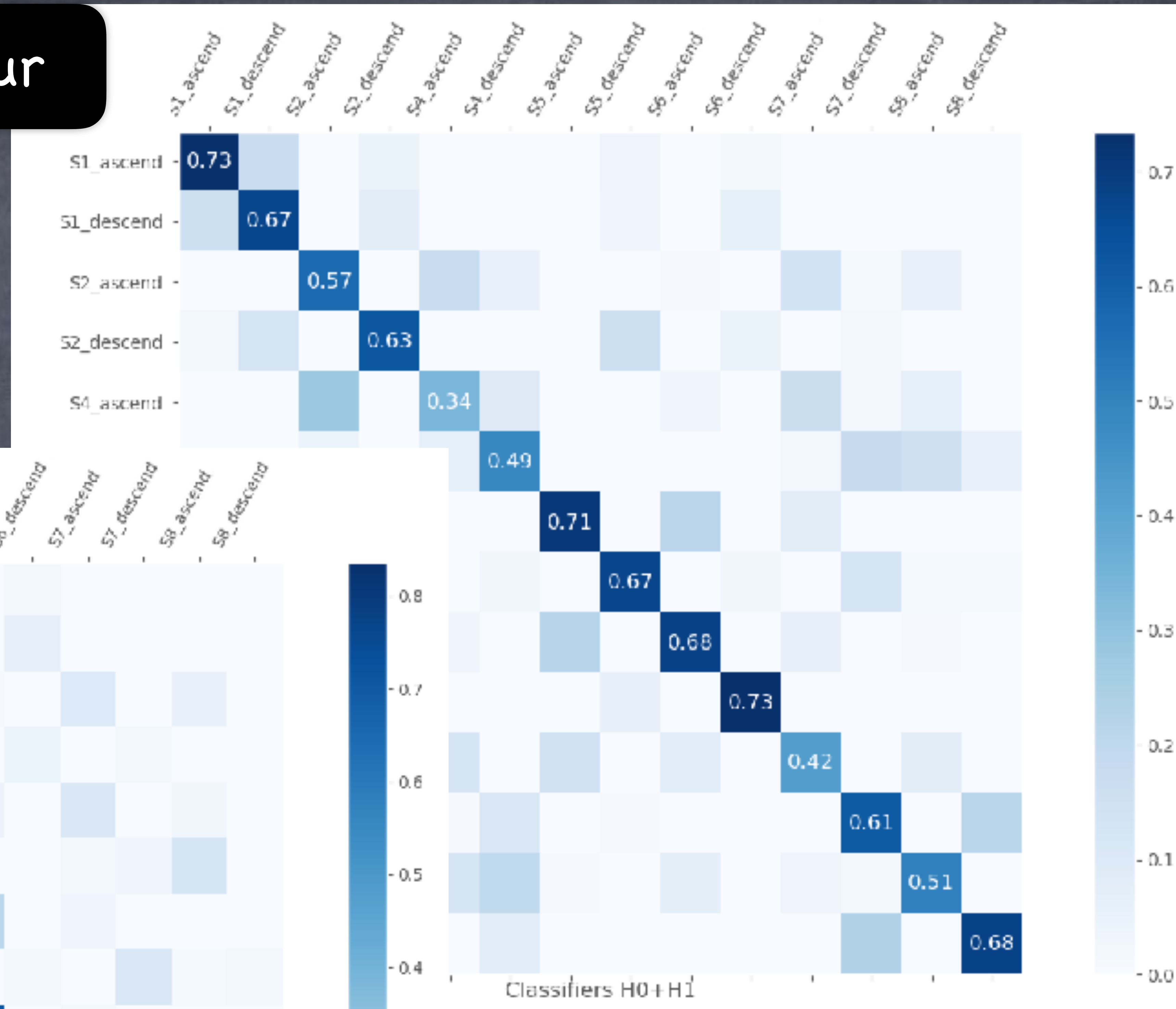

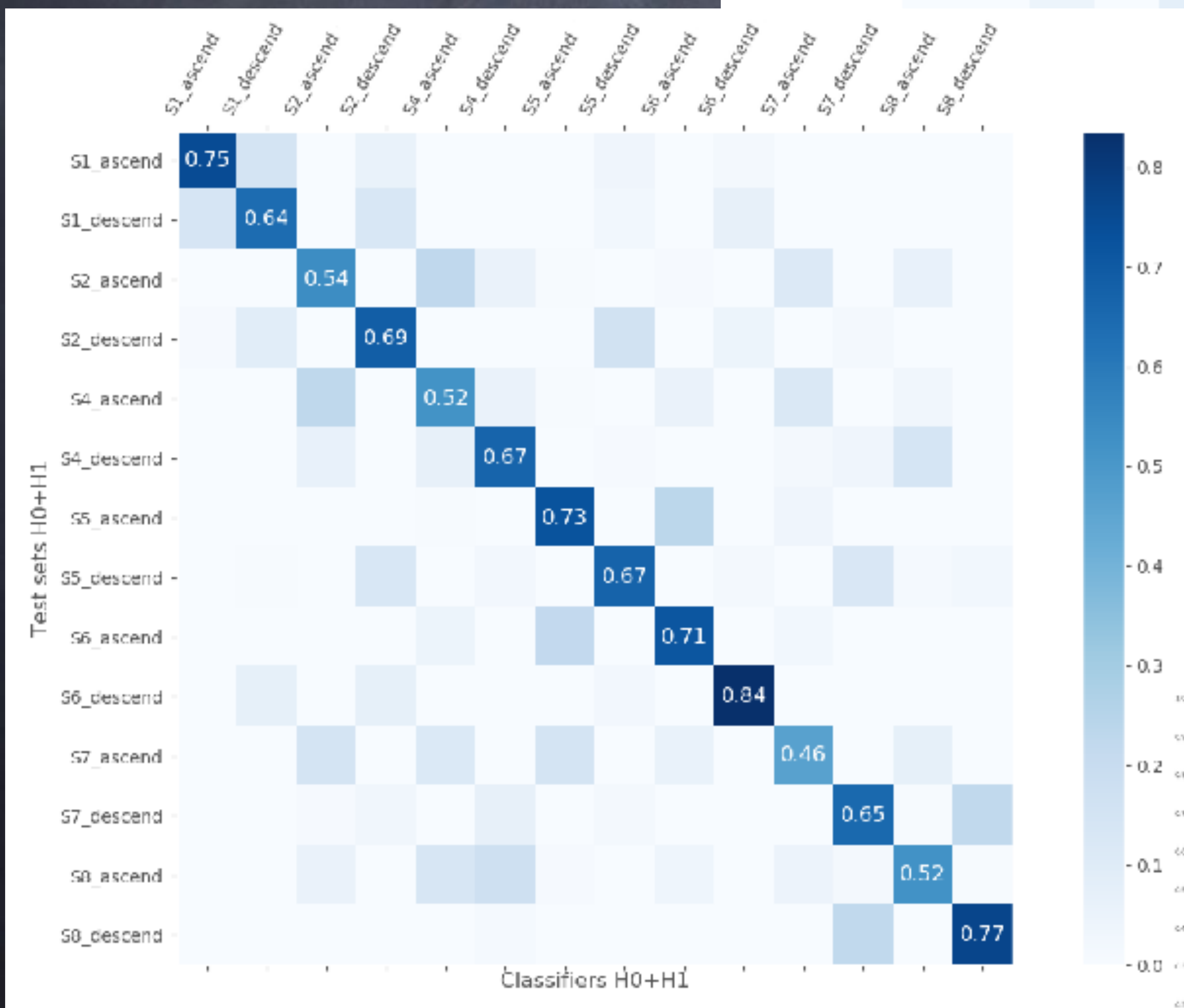

Thank you!