From TDA to functional modeling: spatial cognition

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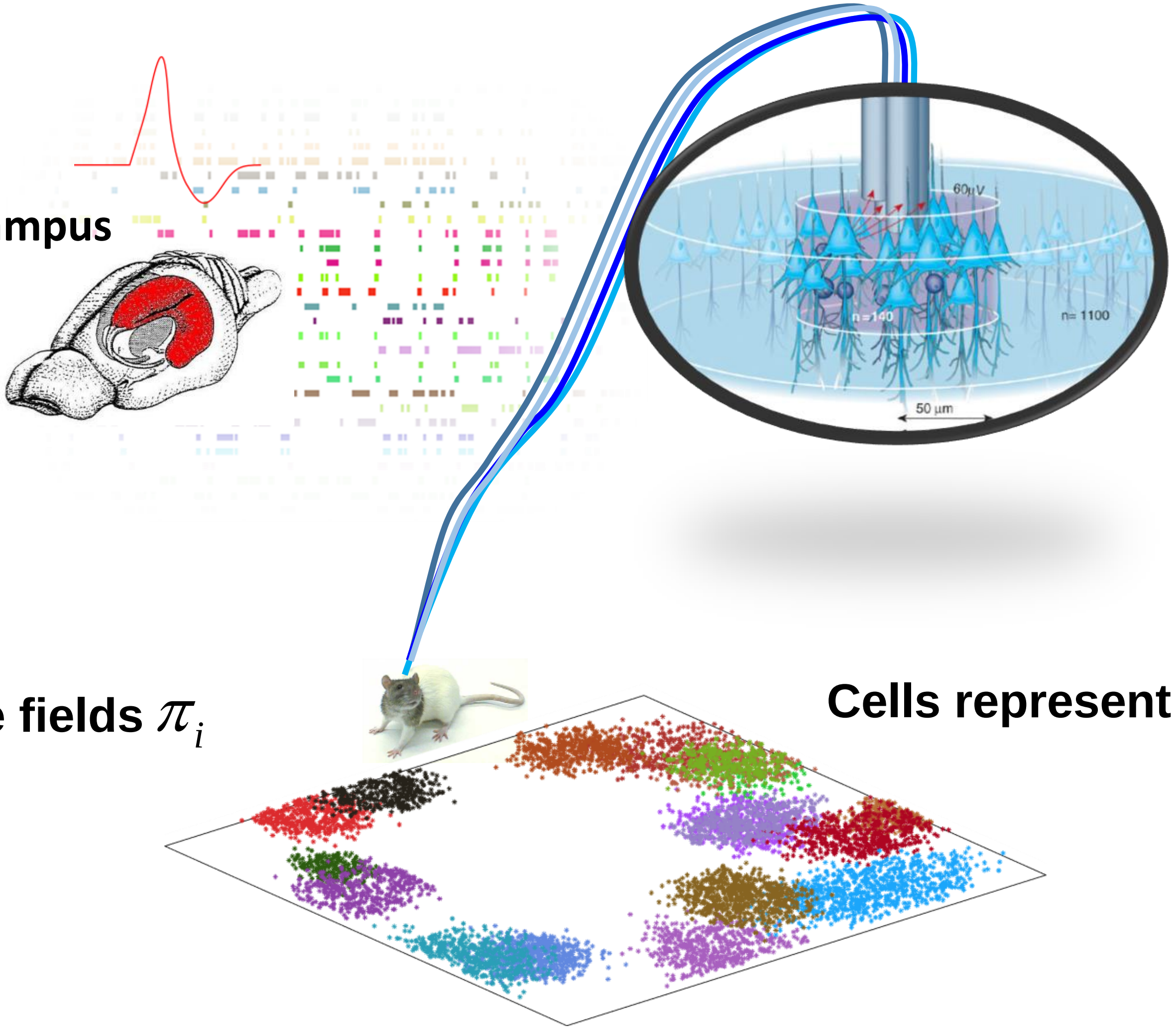
Abel Symposium, Geiranger 2018

Neurons in rodent hippocampus map spatial regions

Hippocampus

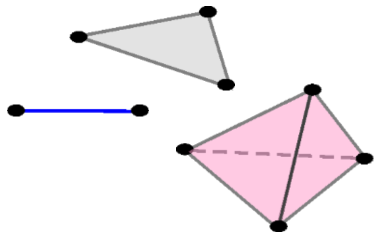


Place fields π_i

Cells represent spatial regions

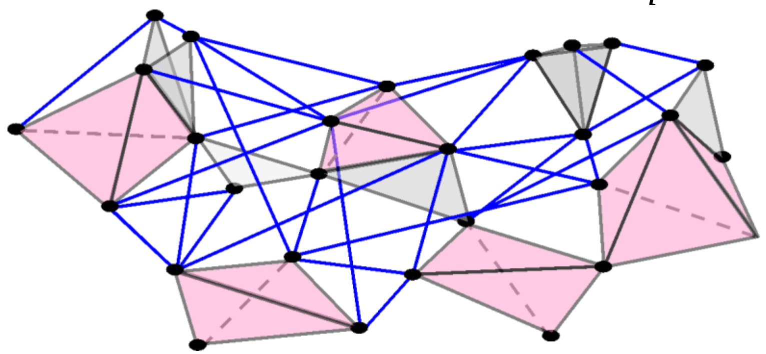
Place field cover → Čech's theorem

Schematically:



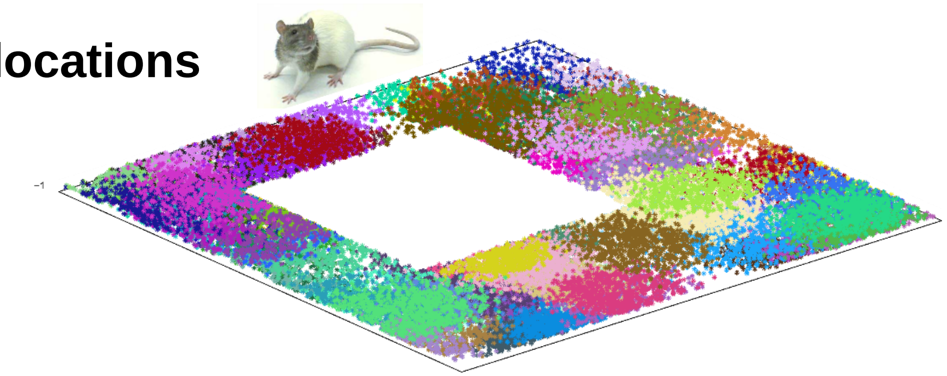
$$\sigma_{ij\ldots k} = \left[\pi_i, \pi_j, \ldots, \pi_k\right]$$

Nerve of the cover, $\mathcal{N} = \bigcup_i \sigma_i$



$$E = \bigcup_i \pi_i$$

A map of locations

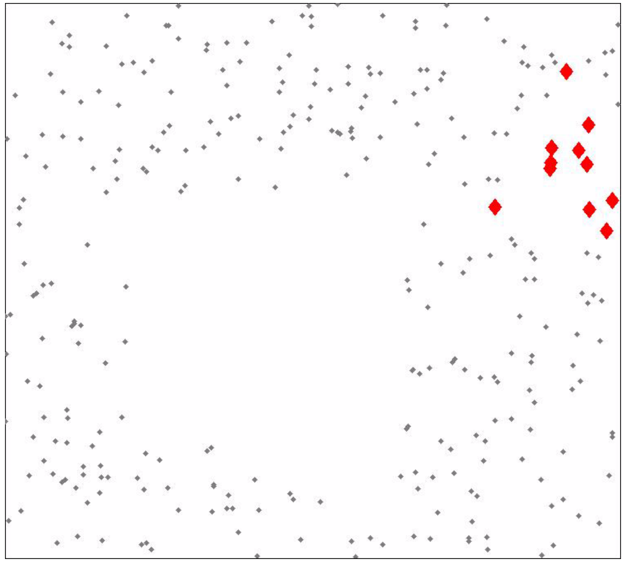


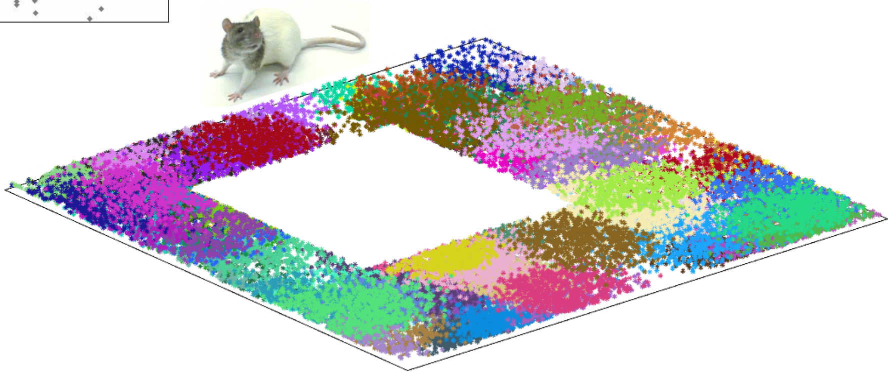
$$H_*(E) = H_*(\mathcal{N})$$

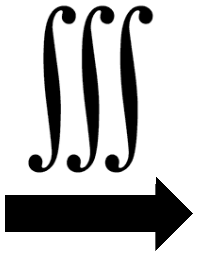
The "nerve" complex $\mathcal{N}$ represents topological information about space $\mathcal{E}$.

P. Alexandrov (1929)
E. Čech (1932)

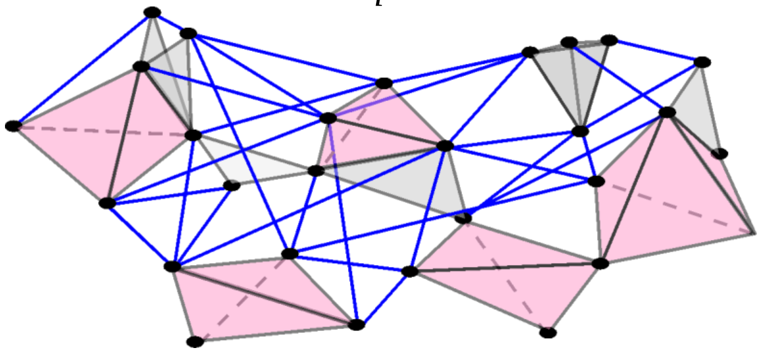
Integrating spiking data into a topological framework







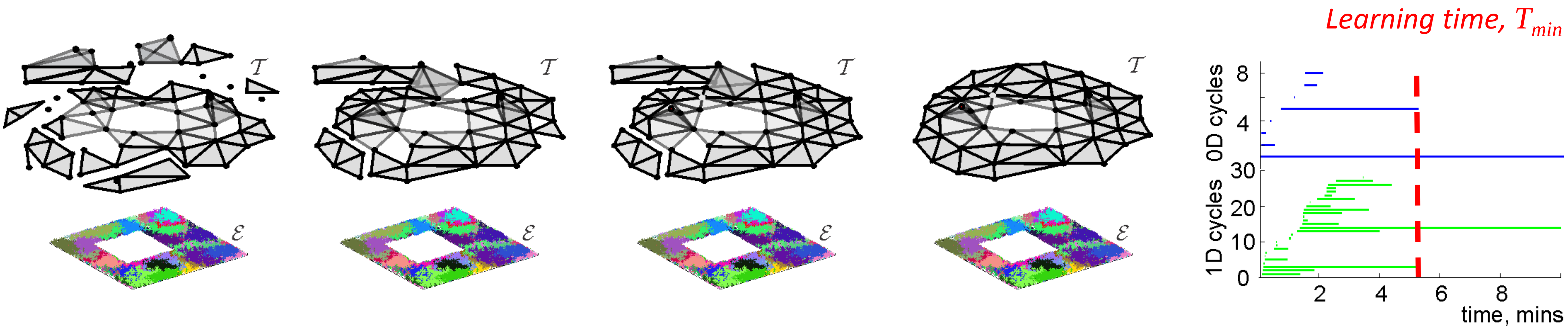
$$\mathcal{T} = \bigcup_i \sigma_i$$



$$\sigma_{ij\ldots k} = \left[c_i, c_j, \ldots, c_k \right]$$

Temporal complex $\mathcal{T}(t)$ also contains the full topological information about space

Growing simplicial complexes: a model of learning



1. Simplexes accumulate, $\mathcal{T}(t_1) \subset \mathcal{T}(t_2)$ for $t_1 < t_2$

2. Complex saturates, $t > T_{\min}, H_*(\mathcal{T}(t)) = H_*(\mathcal{T}(T_{\min})) = H_*(E)$

3. Large-scale, qualitative information about $\mathcal{E}$ emerges from local connections

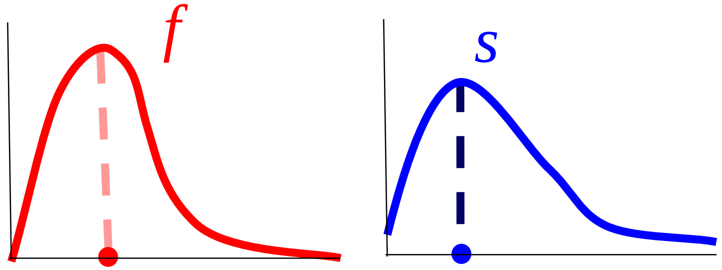
Parameters of place cell activity

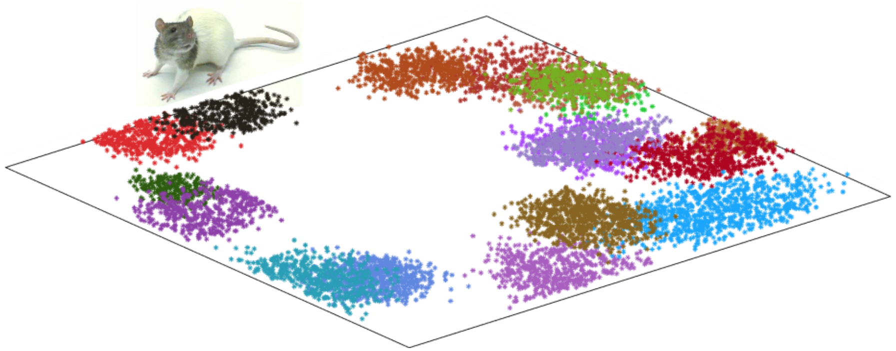
Individual cell's firing rates: $f_1, f_2, \ldots, f_N$

Individual place field sizes: $s_1, s_2, \ldots, s_N$

$$P_f\left(f, \sigma_F\right)$$

$$P_s\left(s, \sigma_s\right)$$

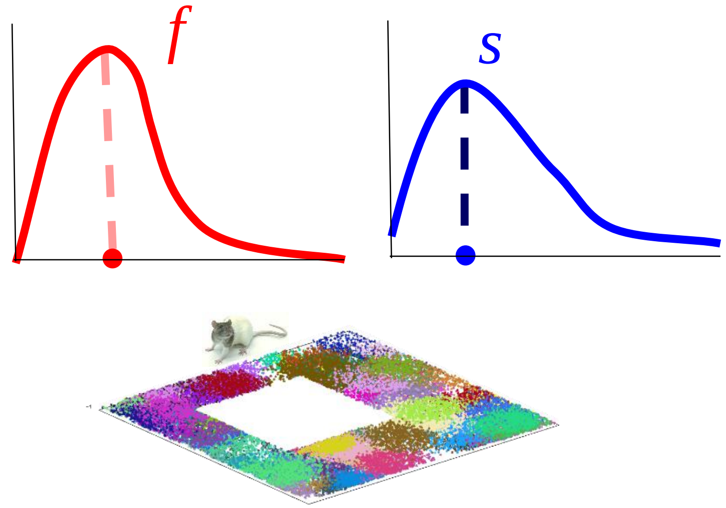


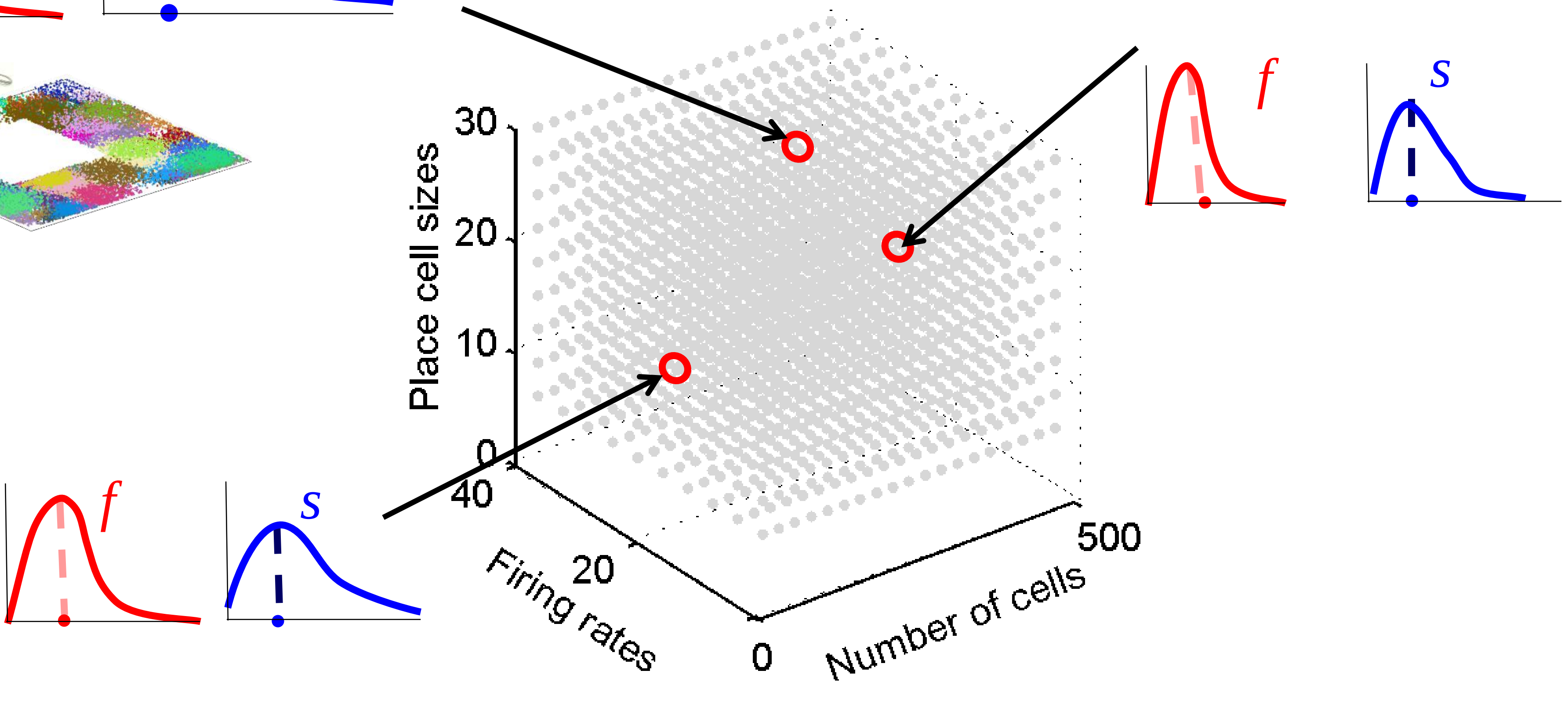


$$T_{min} = T_{min}\left(f, s, N\right)$$

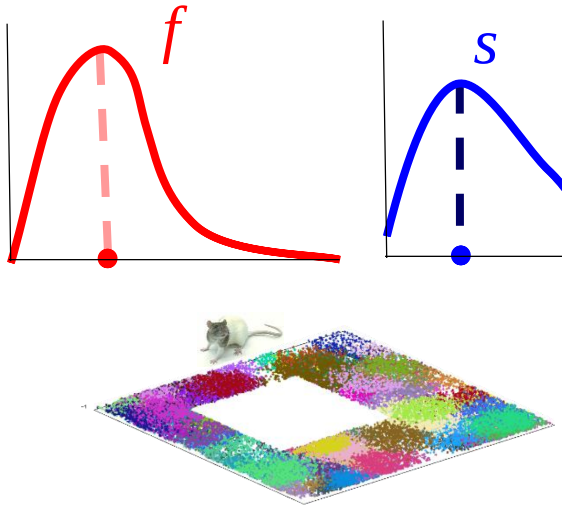
Testing numerically simulated place cell ensembles

Each point (s, f, N) represents a place cell ensemble





Which place cell ensembles produce reliable maps?



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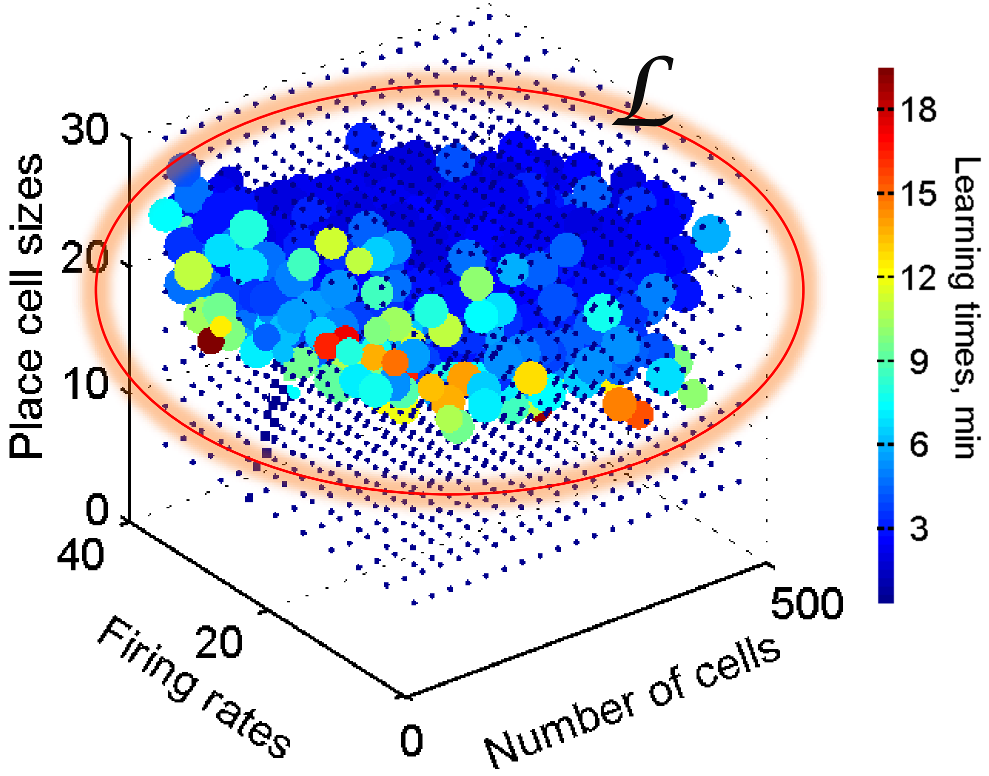
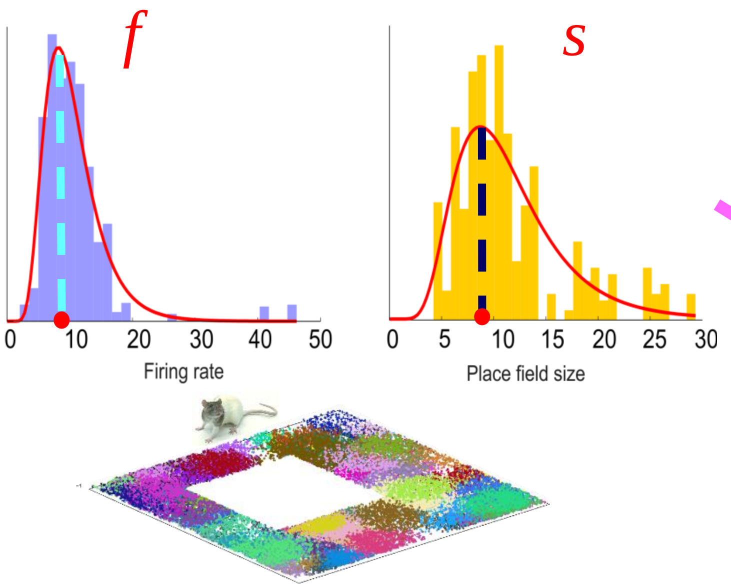
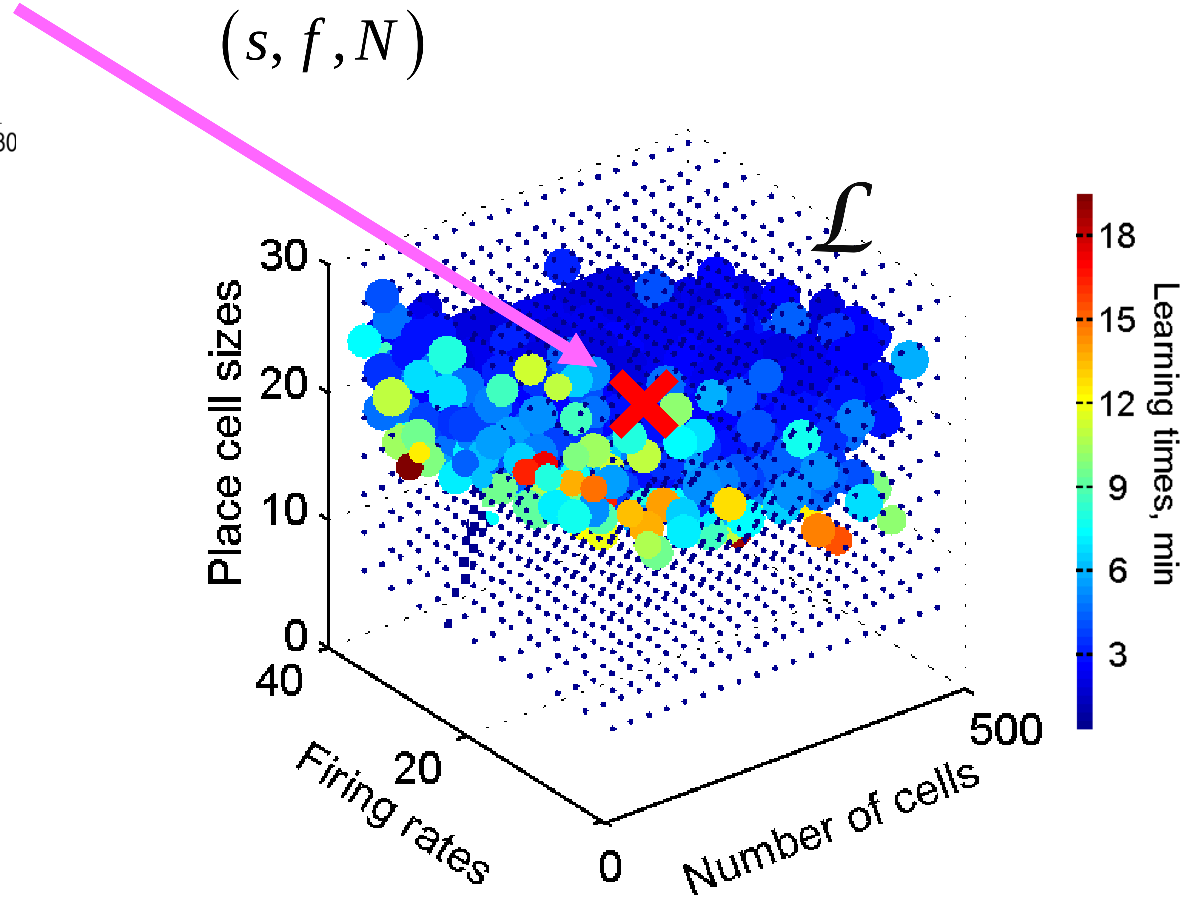


Different neural ensembles acquire information with different efficiencies, depending on firing rate, place field sizes, and size of cell population: the most competent ensembles form the **Learning Region, $\mathcal{L}$**

Experimental parameters fall into learning region





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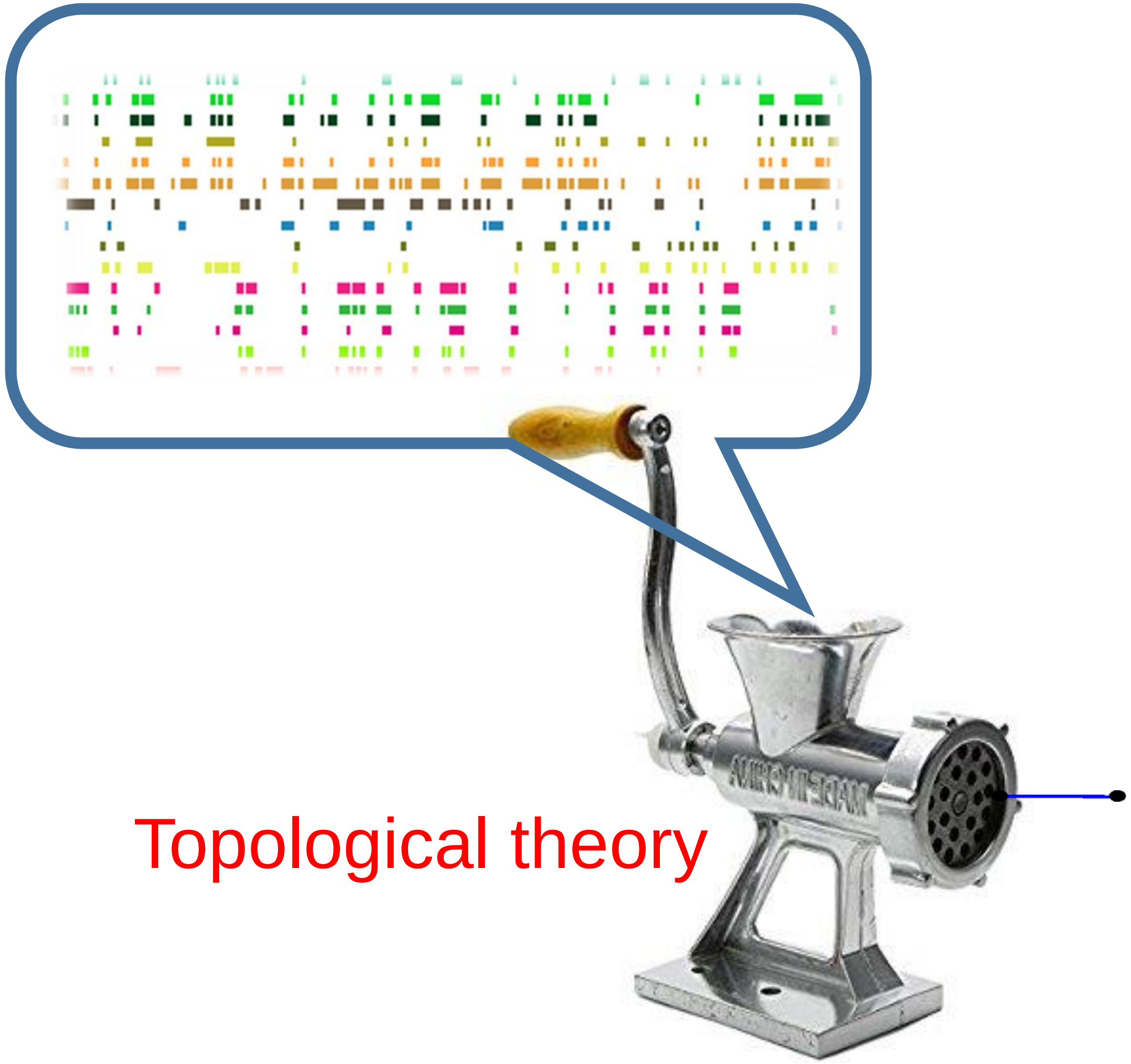
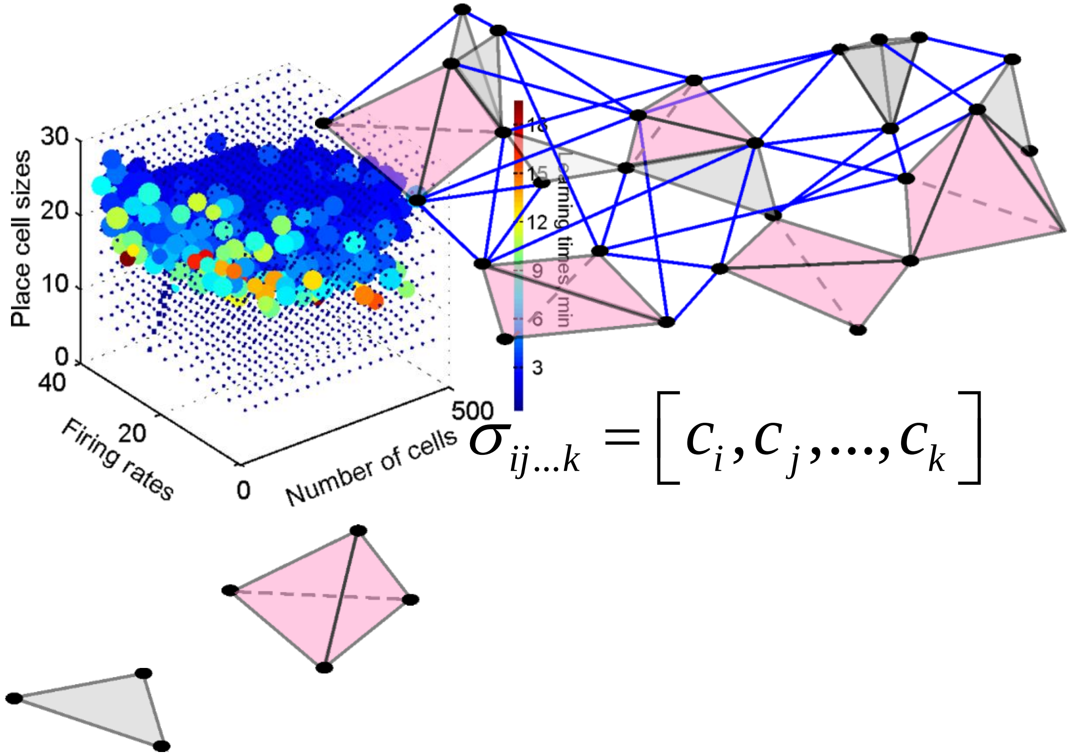
Electrophysiological data



Topological theory

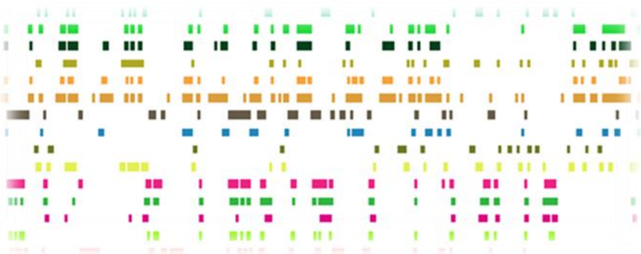
TDA



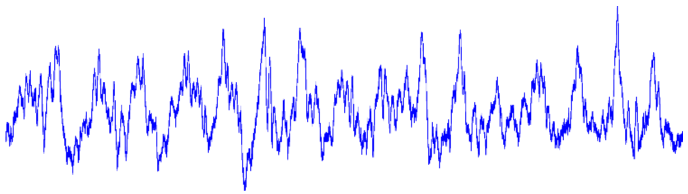
$$\sigma_{ij\ldots k} = \left[c_i, c_j, \ldots, c_k \right]$$

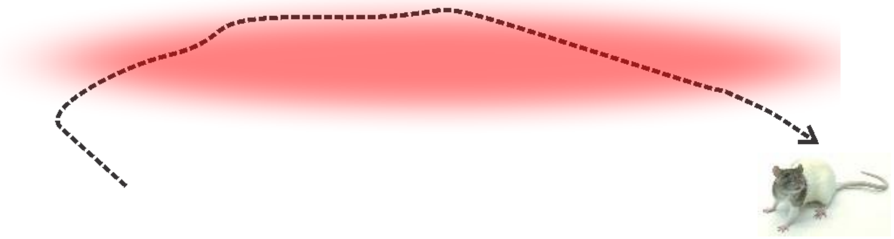
Contribution of other physiological parameters

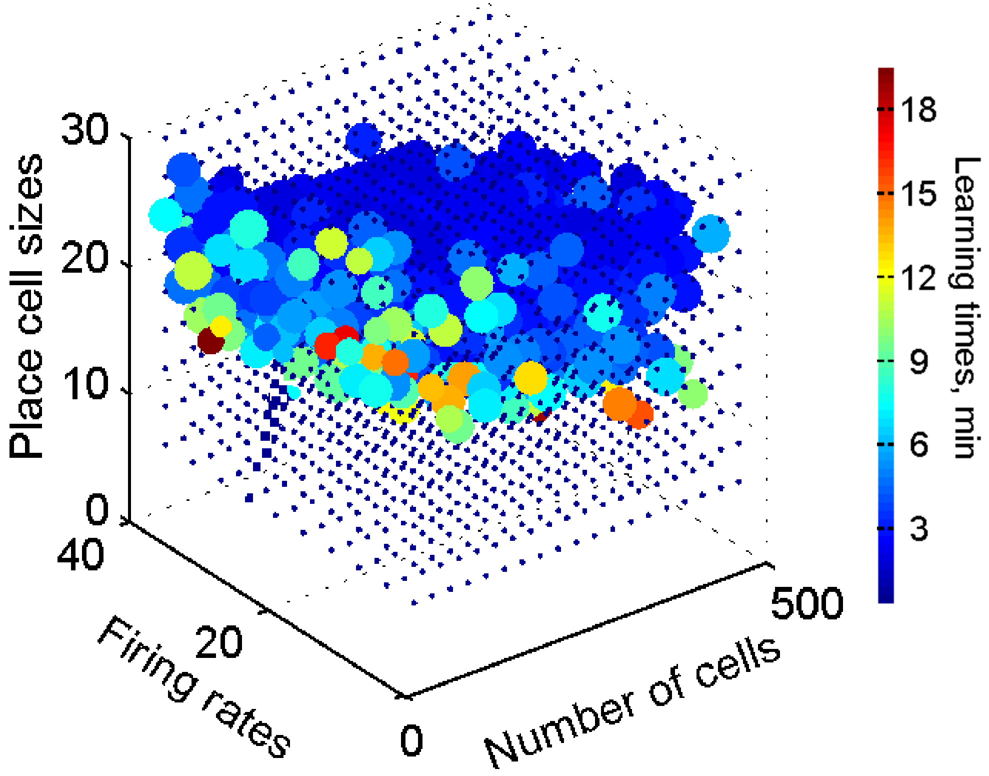
Spike trains



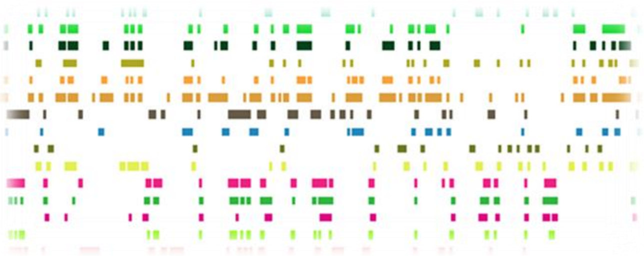
Brain waves





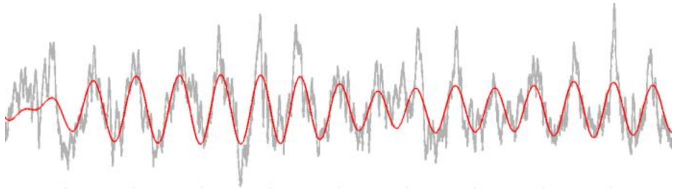


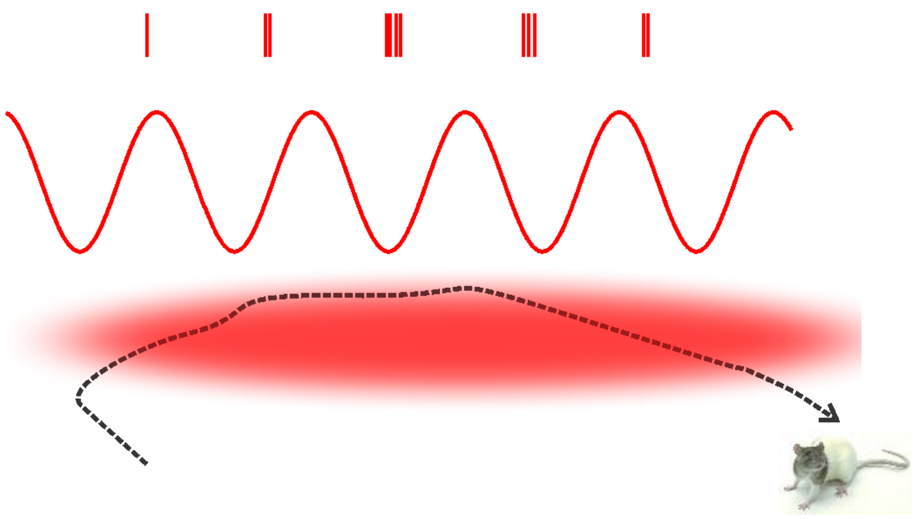
Spike trains

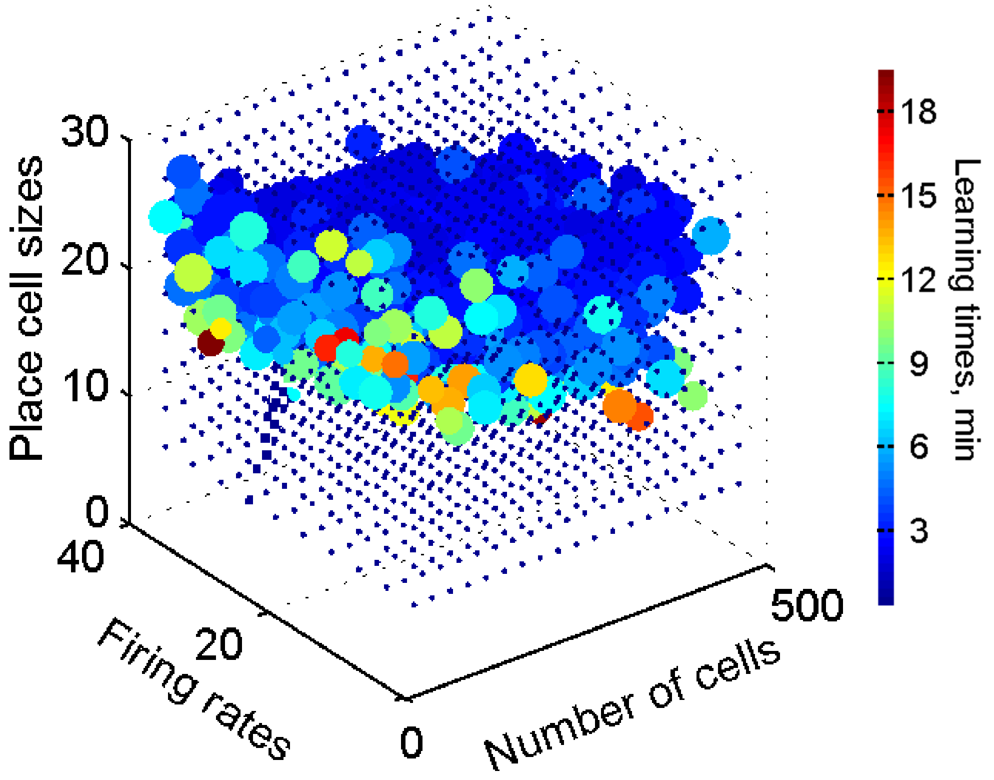


+

Theta wave (4-12 Hz)

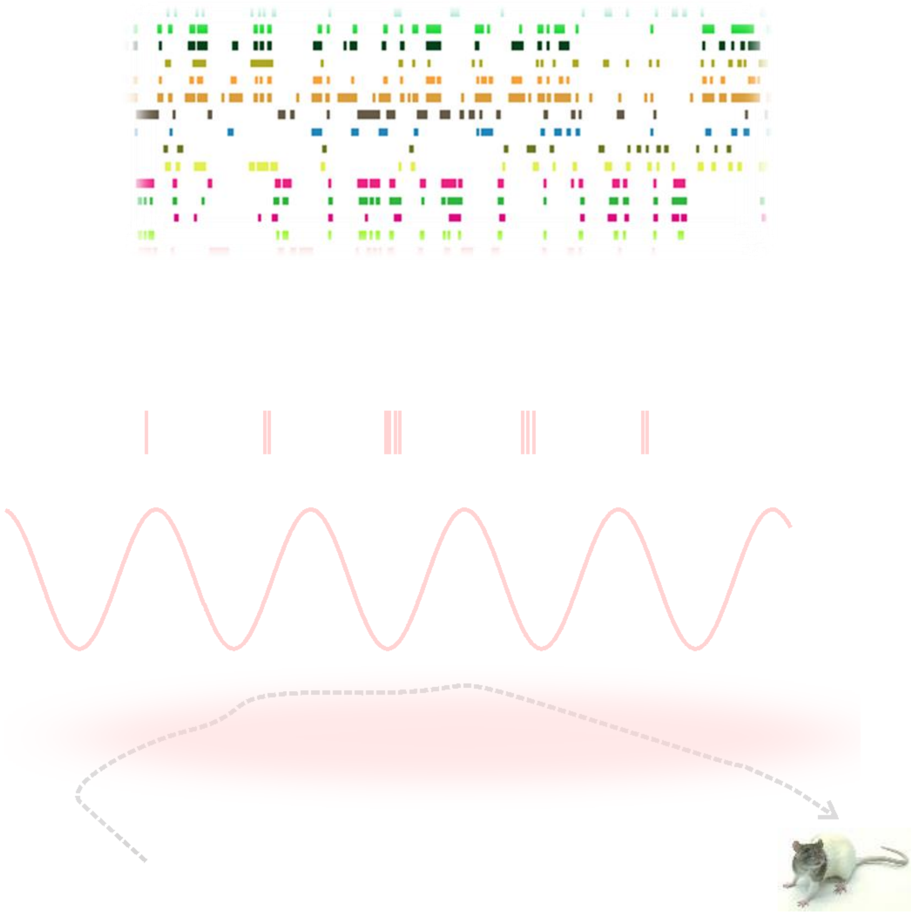




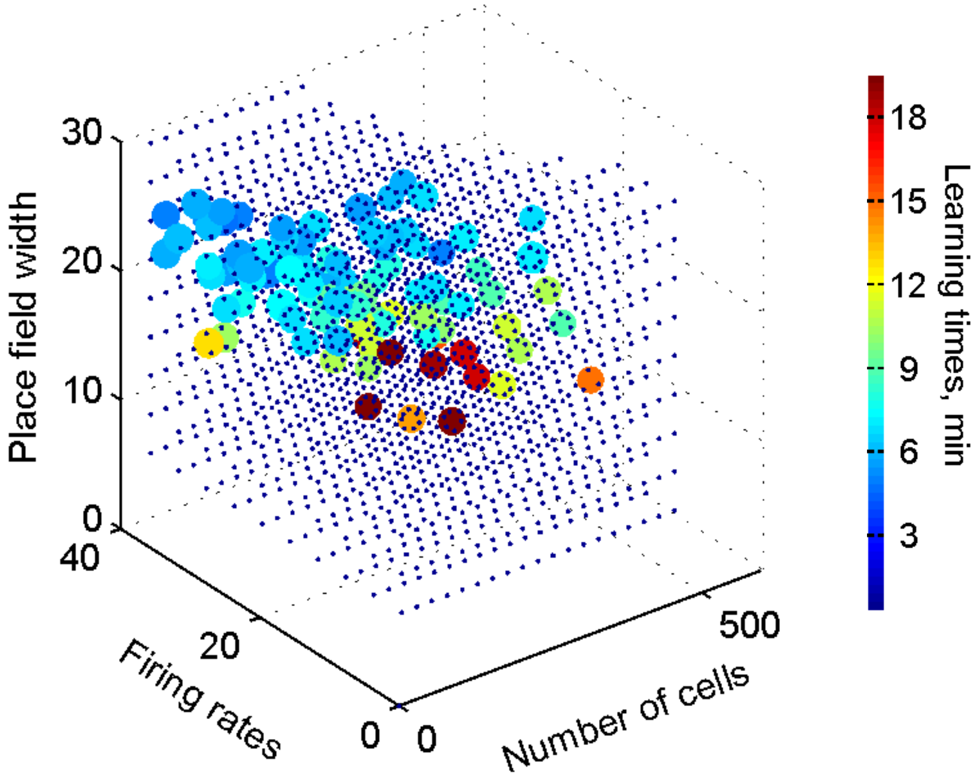


θ-wave modulation is essential for successful learning

Spike trains

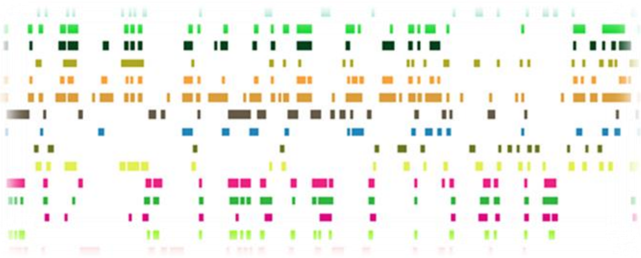


No θ wave



θ-wave modulation is essential for successful learning

Spike trains

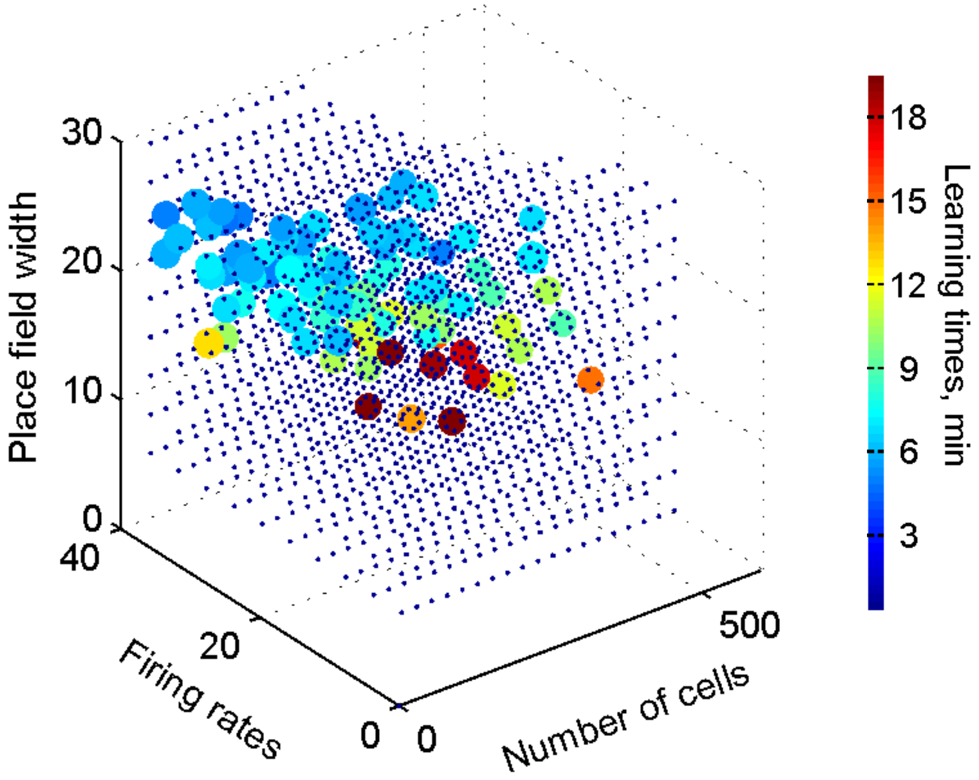


No θ wave

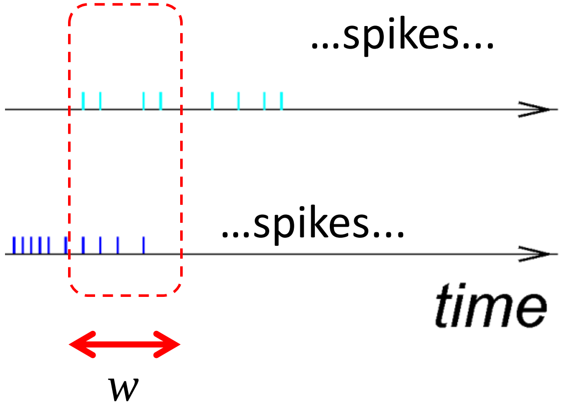
Without θ-wave:

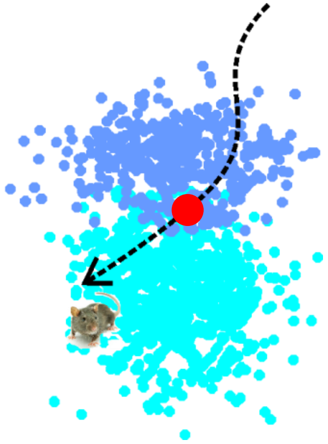
1. Learning times T_{min} increase by >50%
2. Statistical variability of T_{min} increases

3. The spurious loops' lifetimes increase by ~50%
4. Their ***mean*** number decrease by ~30%
5. Their ***maximal*** number increases by ~50%
6. The size of the coactivity complex increases

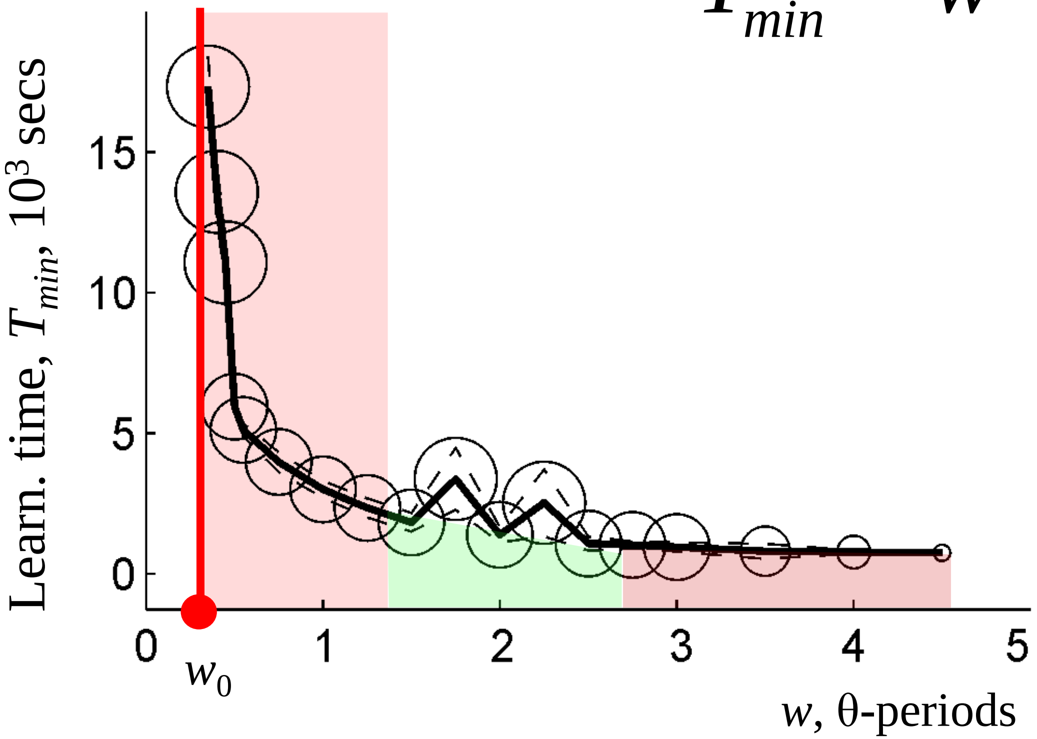


Place cell coactivity detection





$$T_{min} \sim w^{-\alpha}$$



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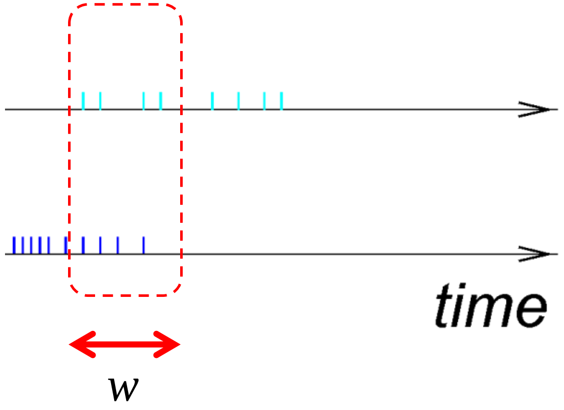
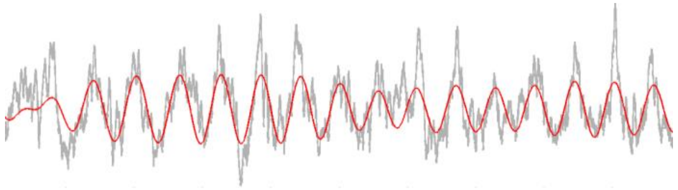
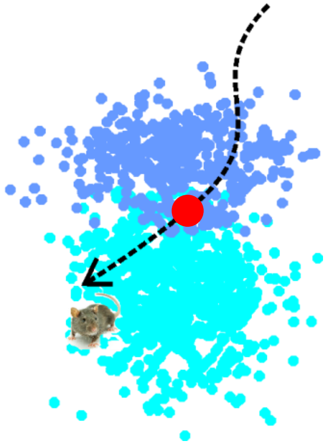
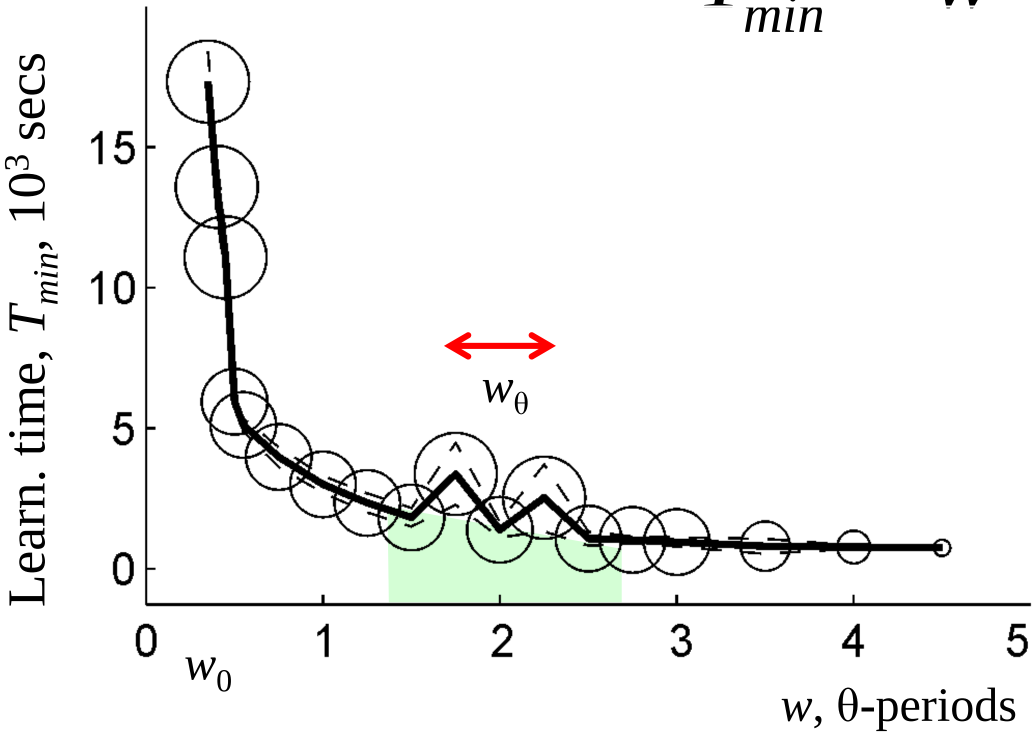
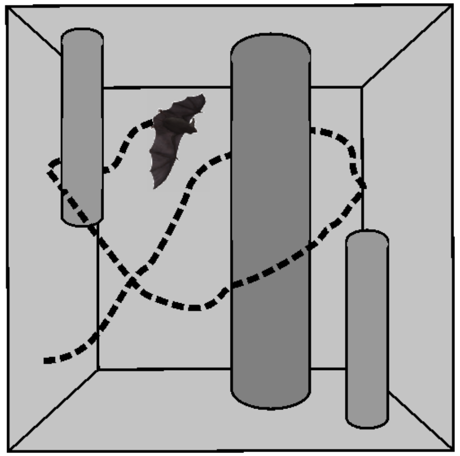


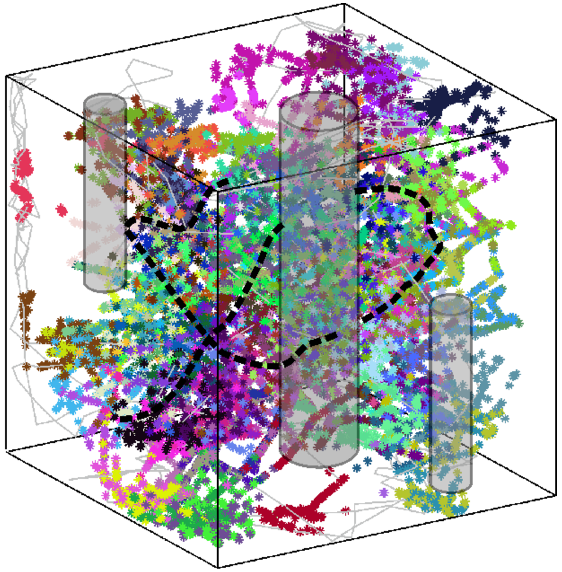






θ-modulation in rats and vs. no θ-modulation in bats



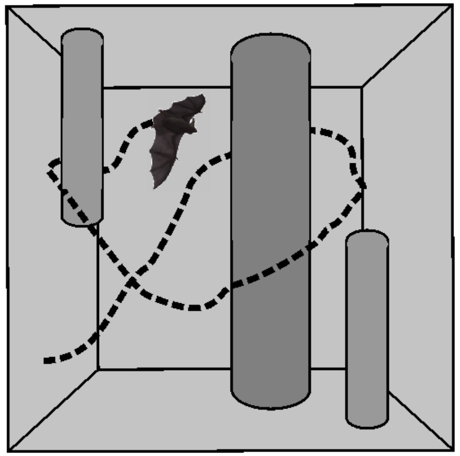


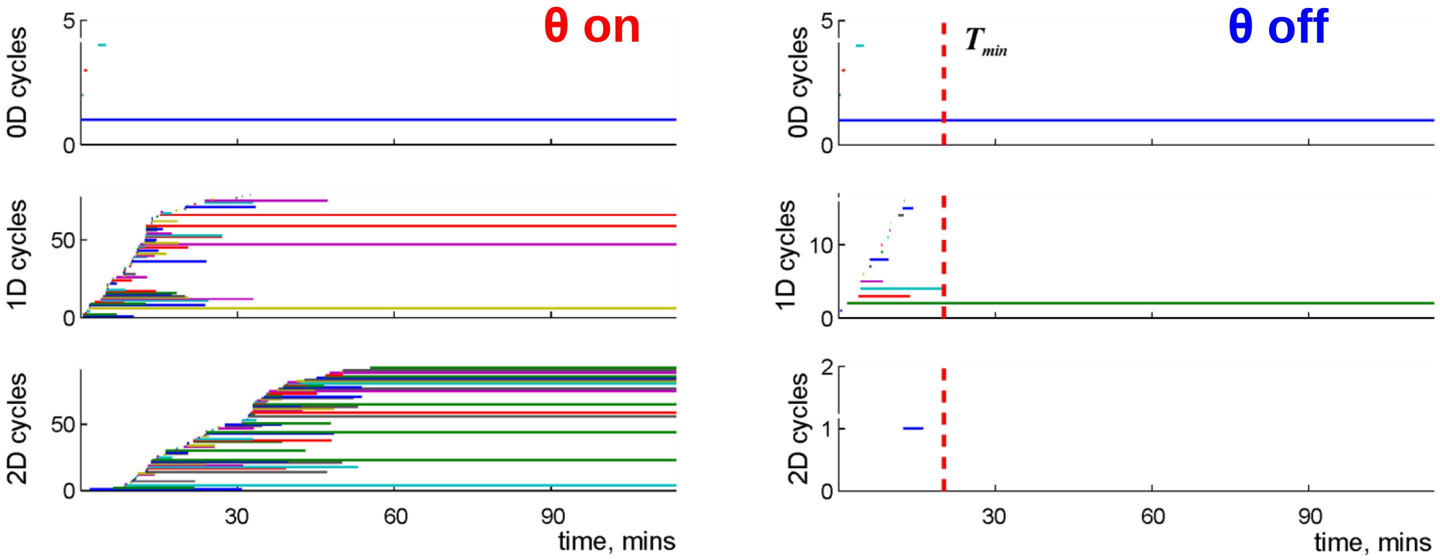
Cognitive map in bats:

3D place fields, as expected

…but no θ-modulation

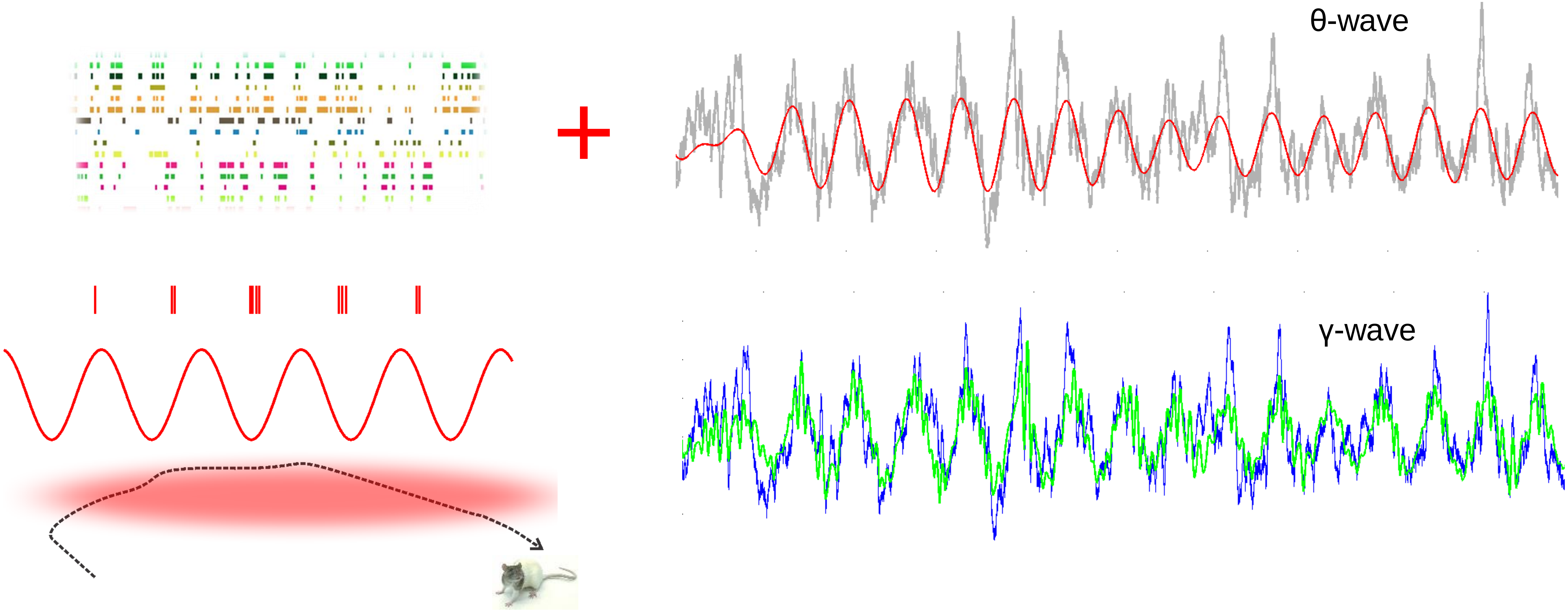
θ-modulation in rats and vs. no θ-modulation in bats



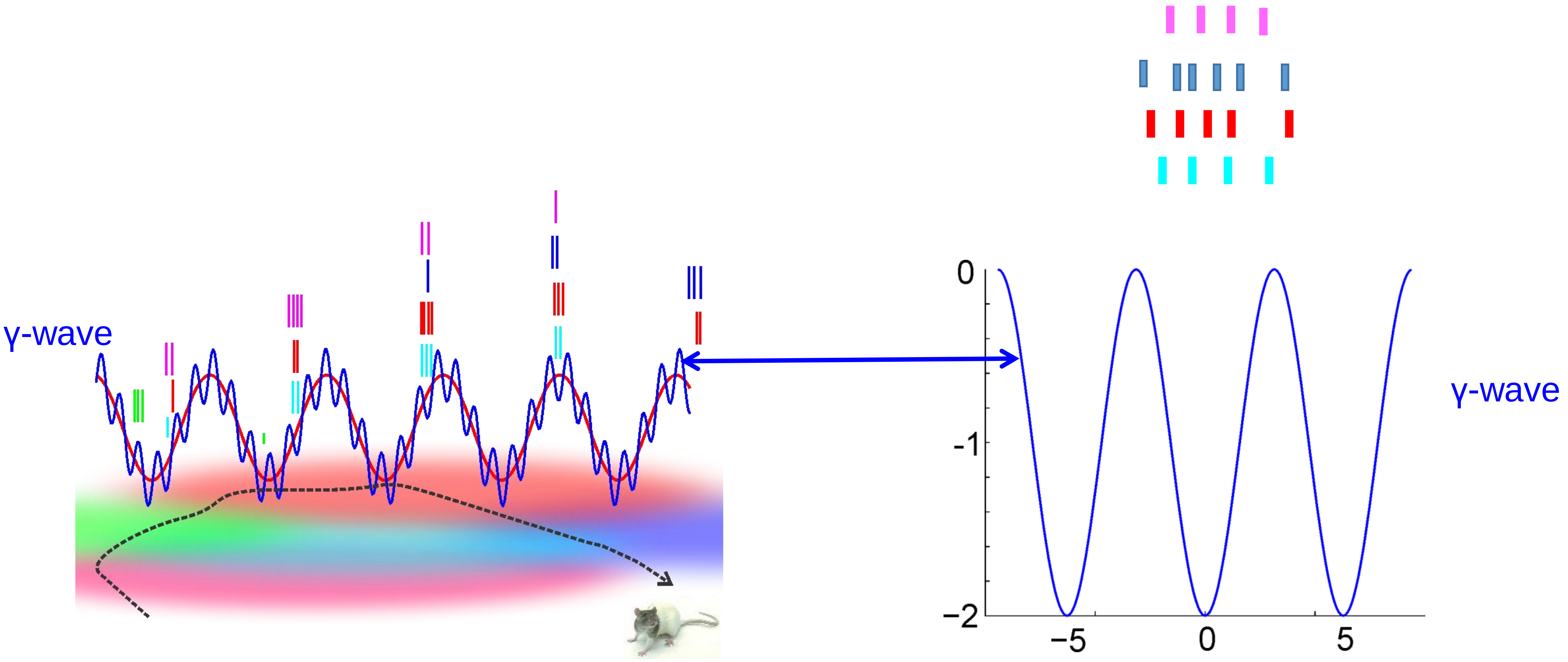


θ-modulation in bats amplifies topological noise in 3D

θ-modulation of spiking activity



γ-modulation of spiking activity



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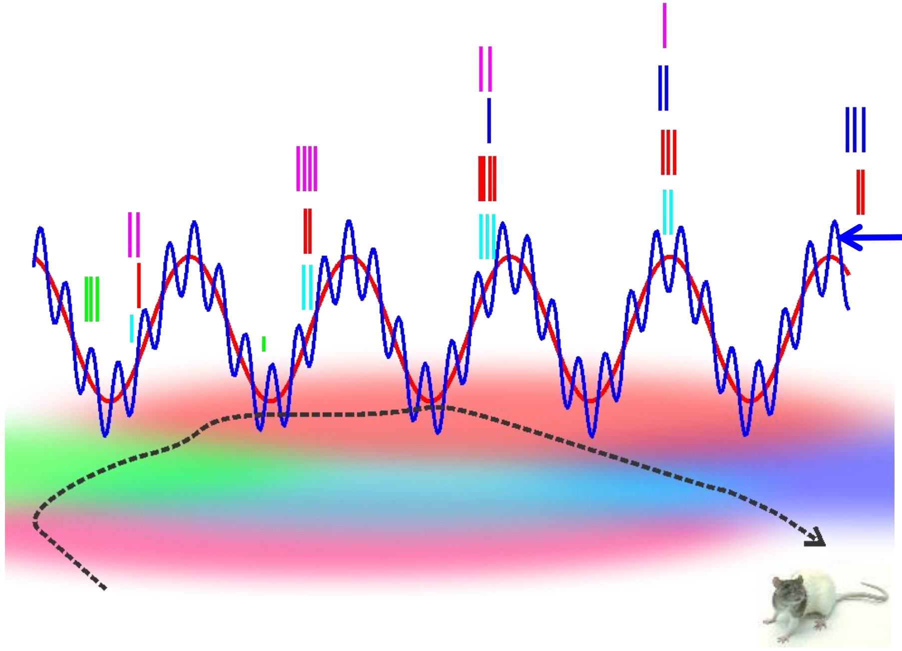
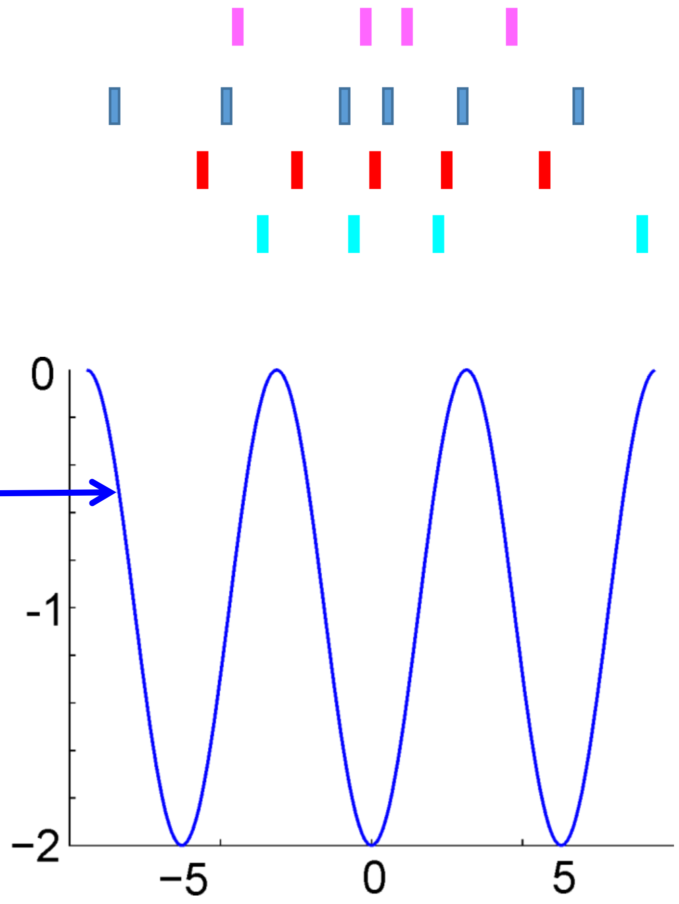
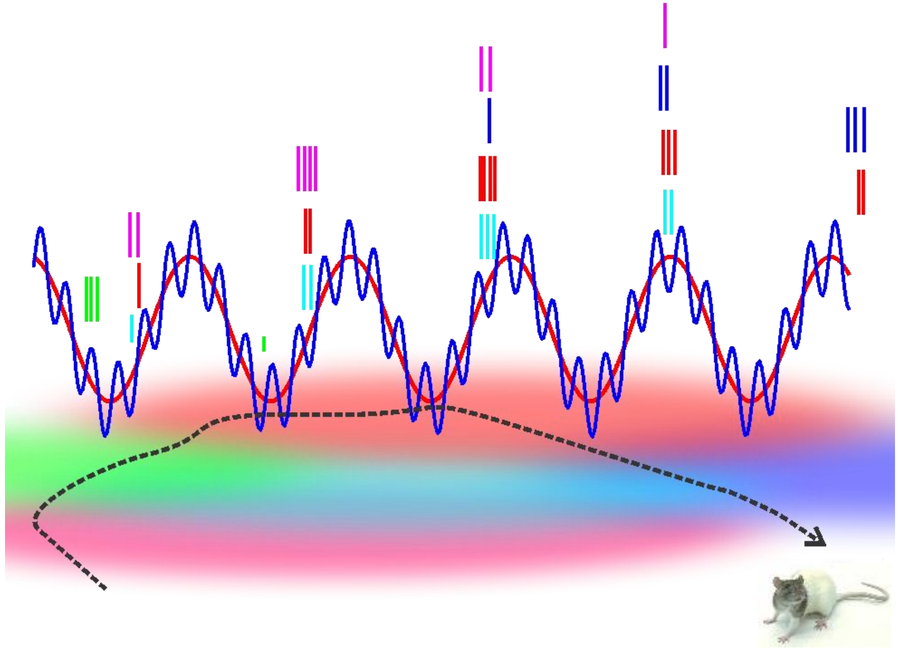


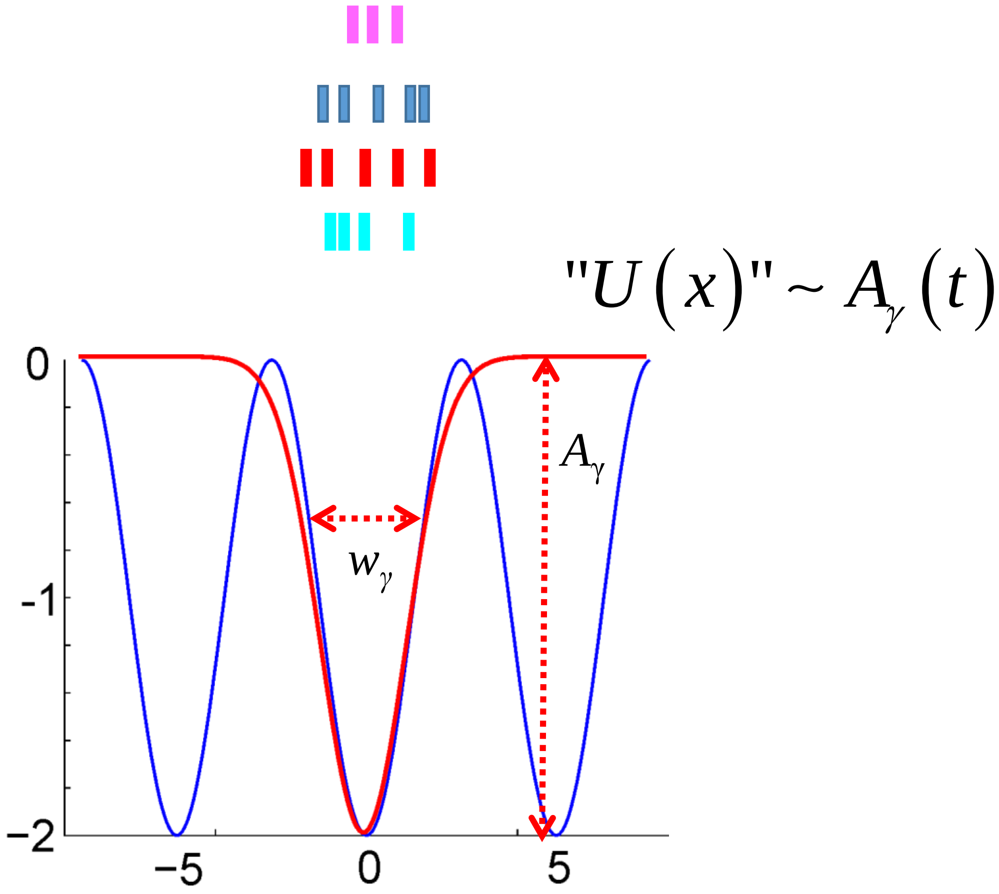


γ-modulation of spiking activity

Effective temperature T (or $\beta = 1/T$)

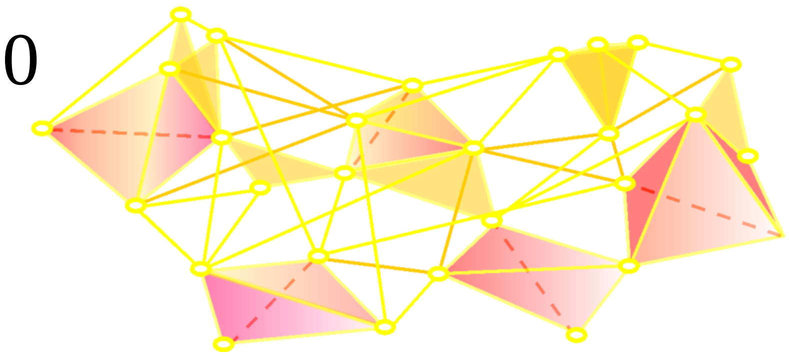
$$p(t-t_i) \sim e^{-A_\gamma(t_i)\beta}$$

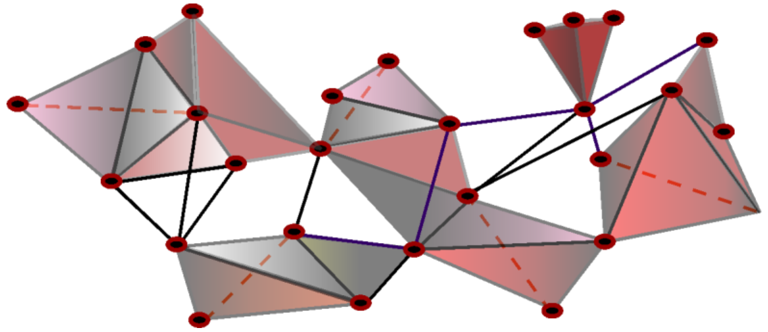




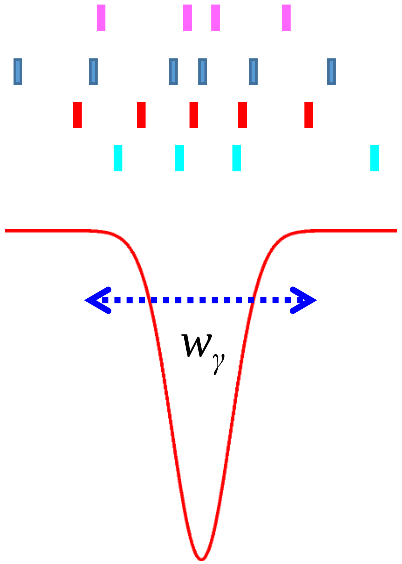
γ-modulation: “hot” vs. “cold” complexes

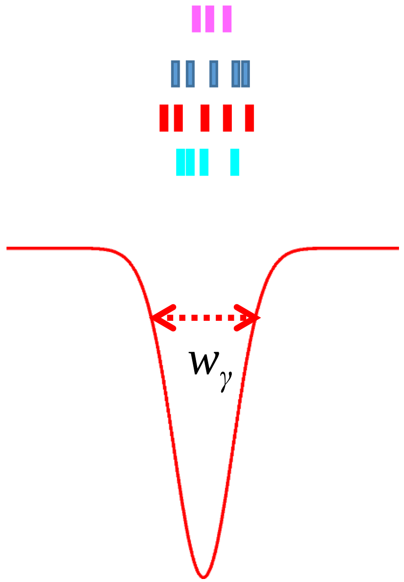
$\beta \sim 0$





$\beta \gg 0$





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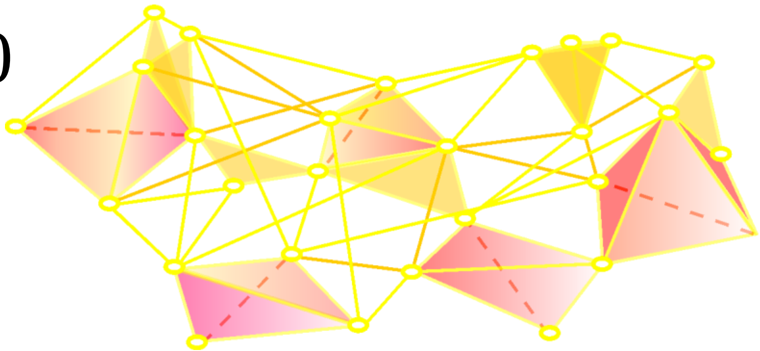
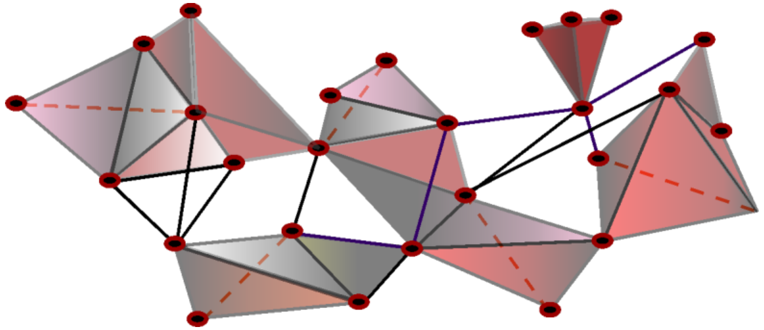
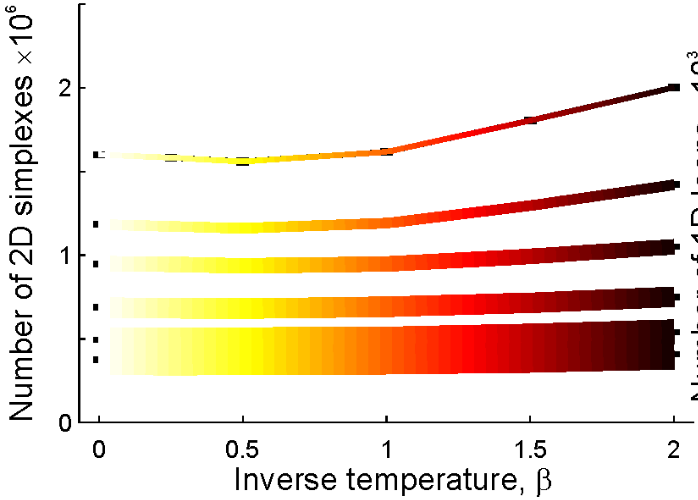
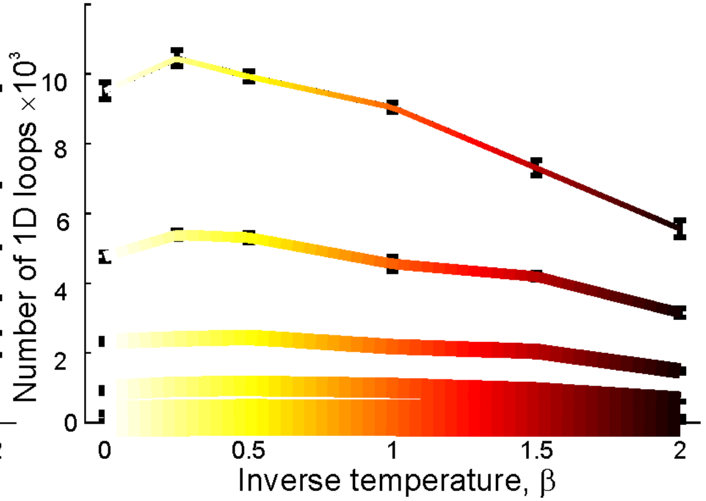


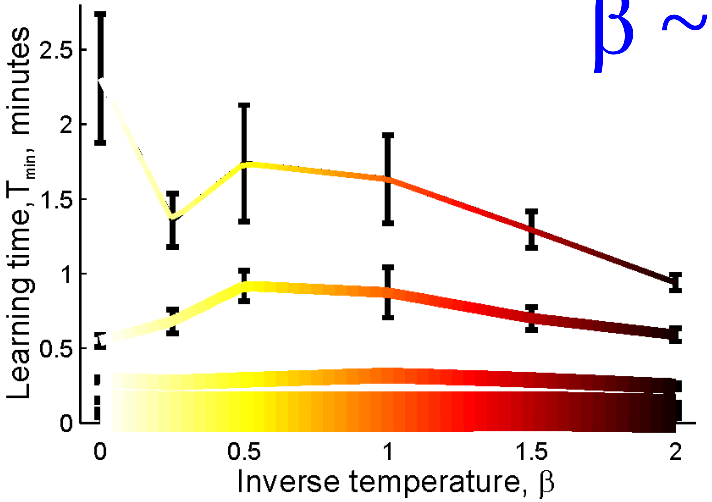


$\beta \gg 0$

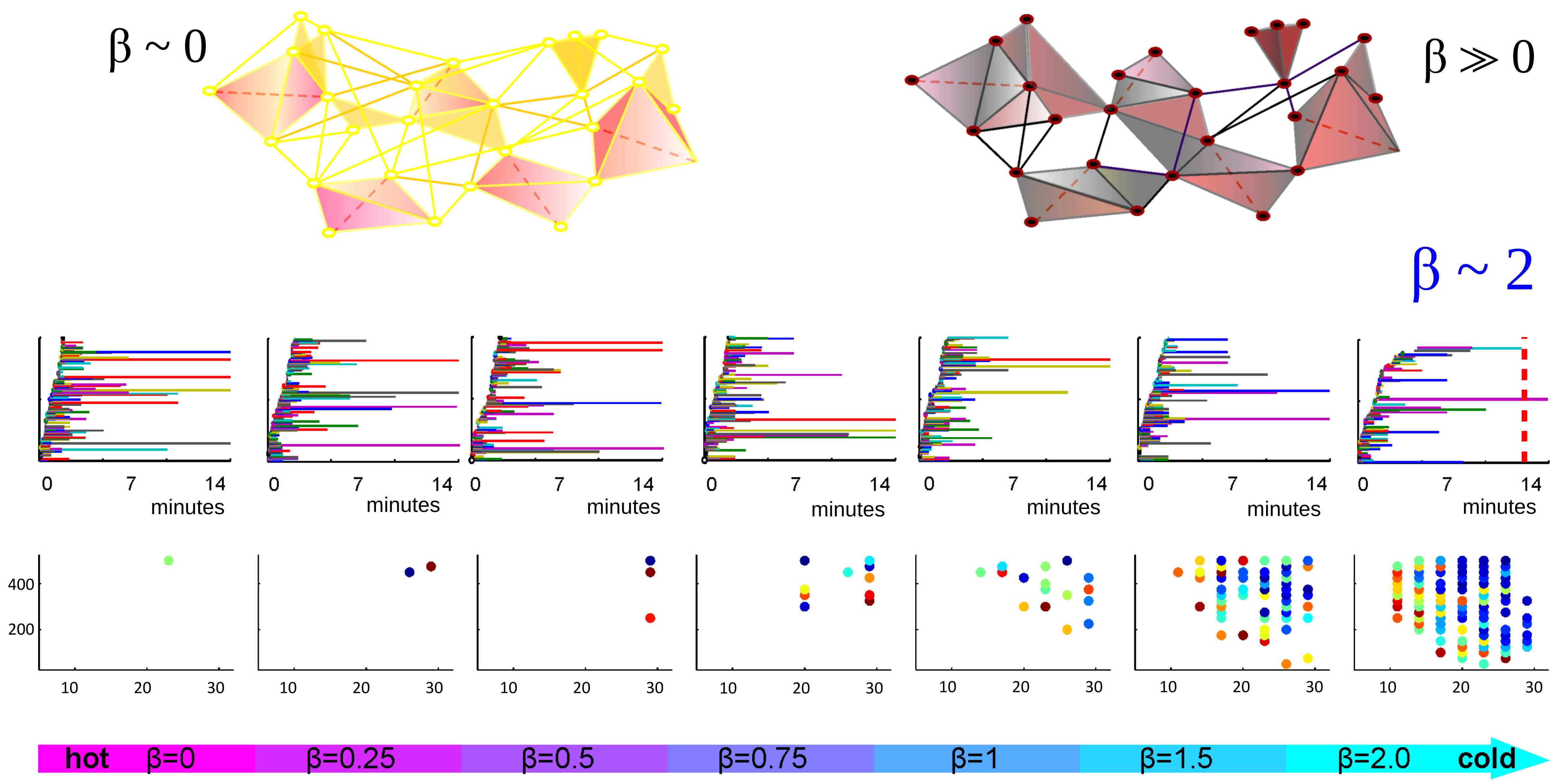
$\beta \sim 2$

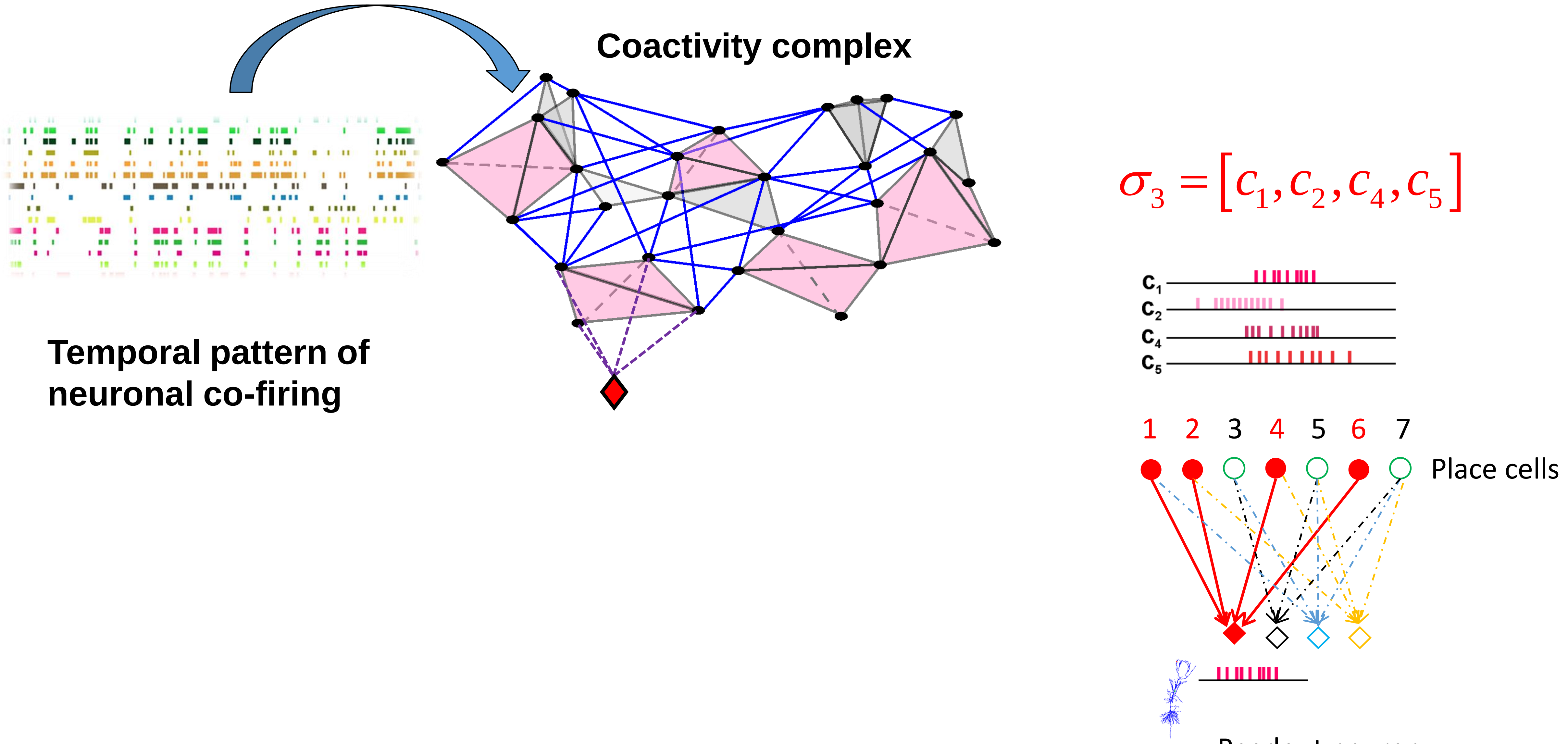


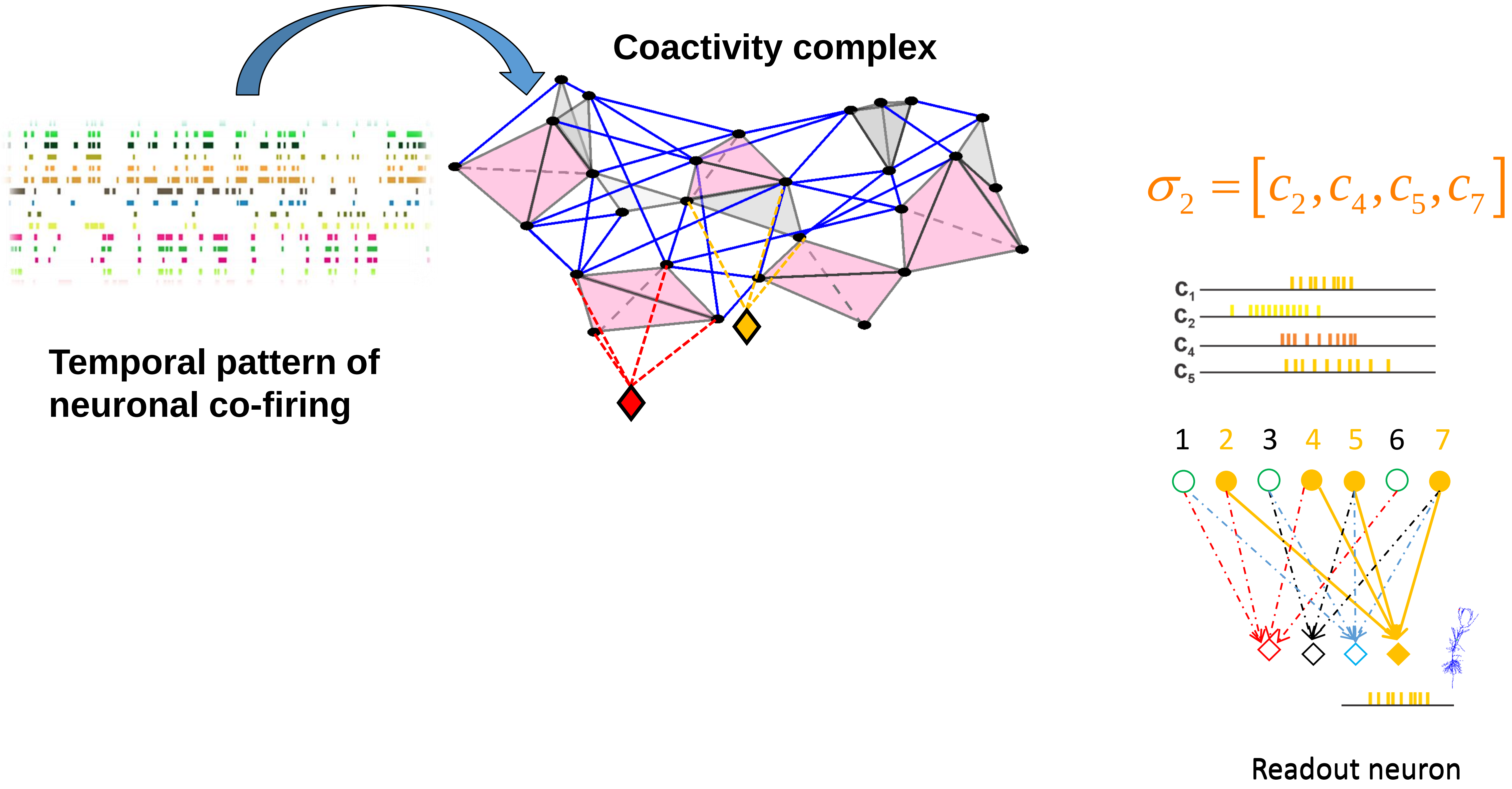




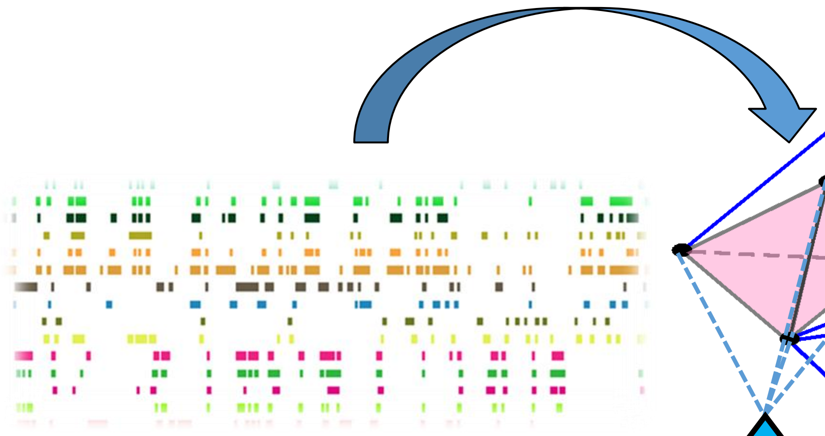
Freezing out topological defects



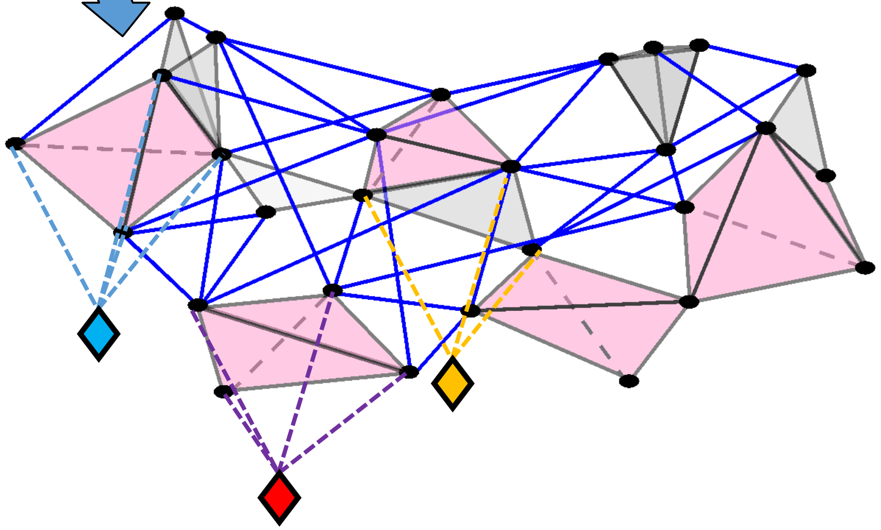

Coactivity complex
Temporal pattern of neuronal co-firing
$\sigma_3 = [c_1, c_2, c_4, c_5]$
c_1
c_2
c_4
c_5
1 2 3 4 5 6 7
Place cells
Readout neuron



Coactivity complex
Temporal pattern of neuronal co-firing
$\sigma_2 = [c_2, c_4, c_5, c_7]$
c_1
c_2
c_4
c_5
1 2 3 4 5 6 7
Readout neuron

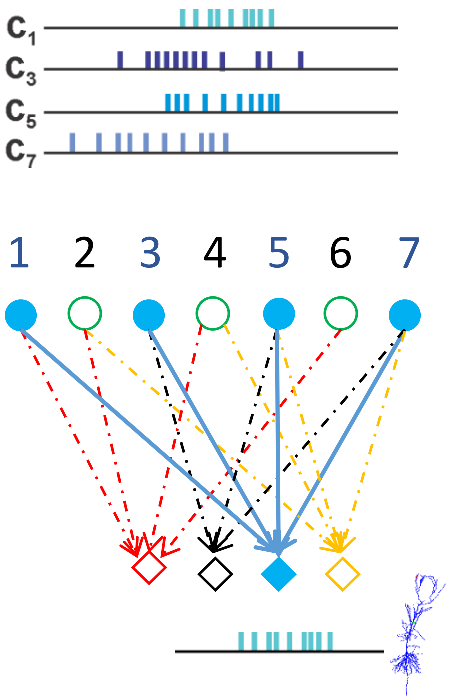


Coactivity complex



Temporal pattern of neuronal co-firing

$$\sigma_1 = [c_1, c_3, c_5, c_7]$$



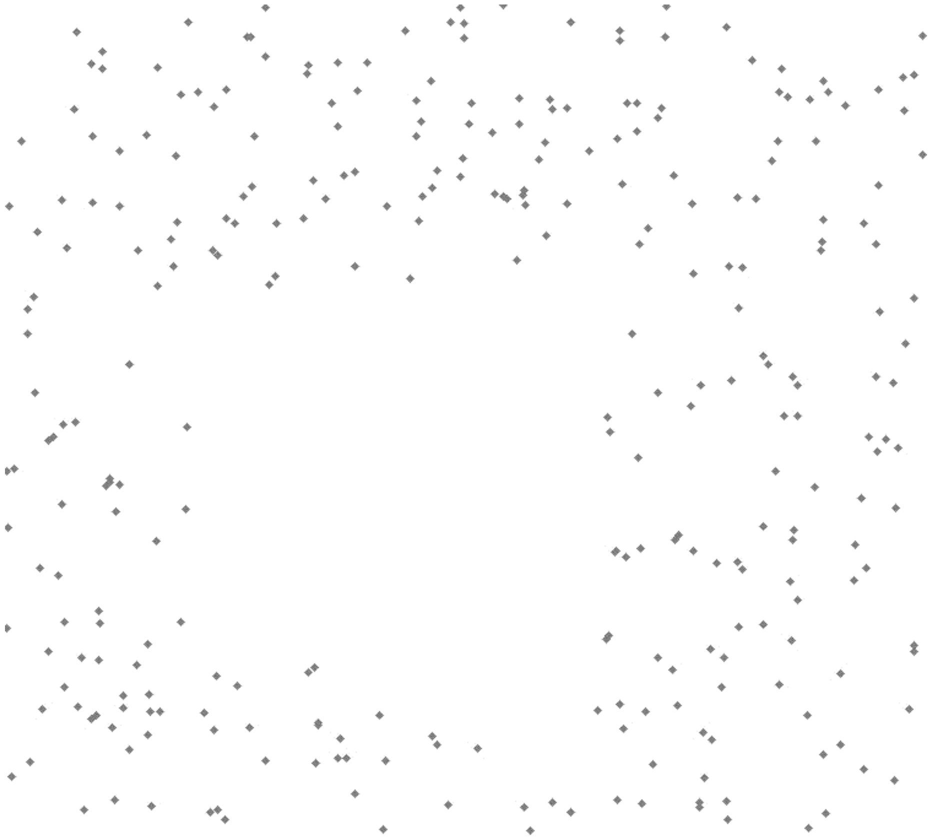
Readout neuron

The pool of coactive place cell combinations is huge, but the number of readout neuron is limited.

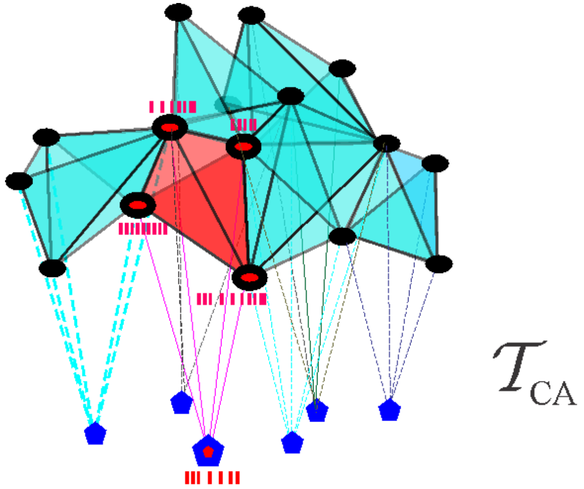
Physiologically: $\#\,coactive\ combinations \sim \binom{\#cells}{\#coactive\ \ cells}$

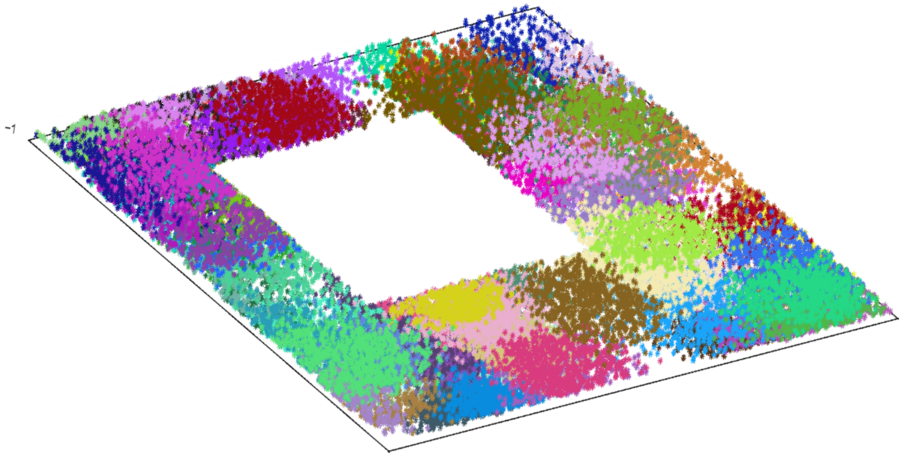
Select the most active combinations of place cells

Selection of cell assemblies based on coactivity rates

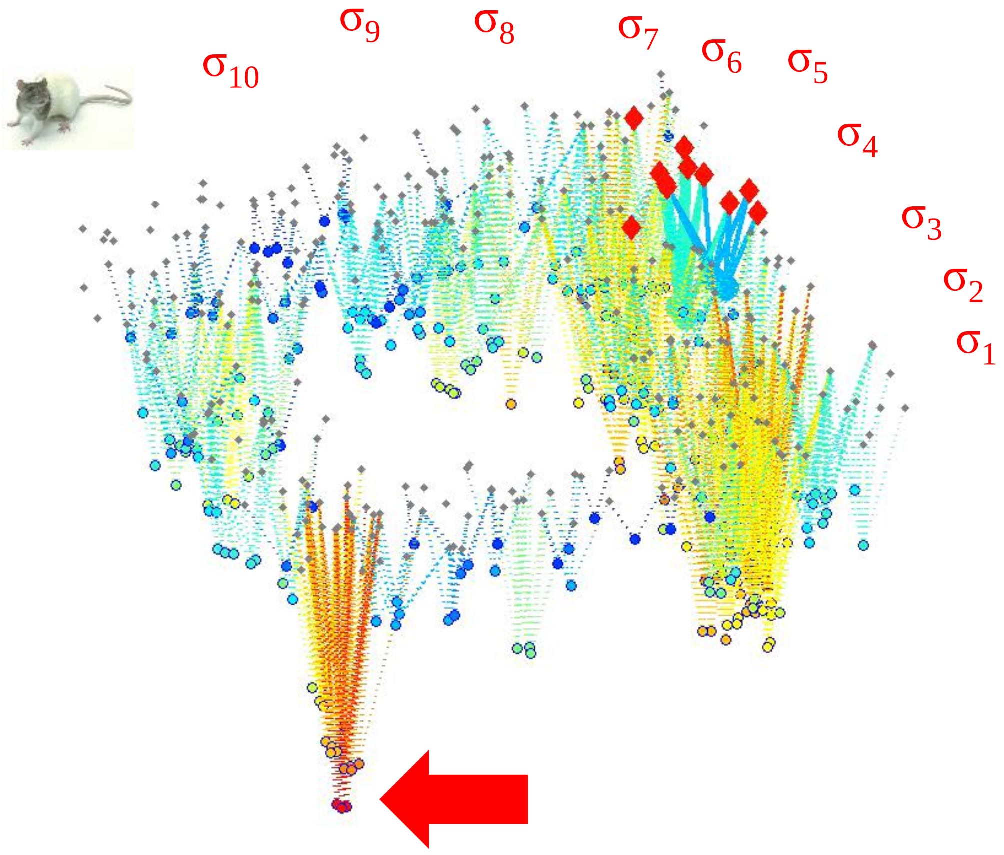


Cell assembly complex



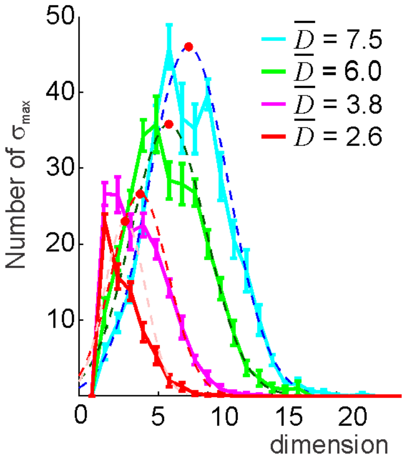


Navigation in cell assembly complex



1. # Cell Assemblies ≈ # place cells, $N_{\max} \approx N_{\mathrm{vtx}}$

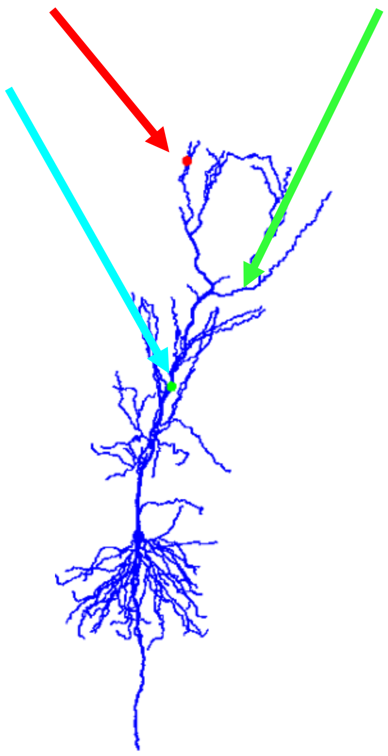
2. Mean contiguity $\xi = \left\langle \dfrac{\dim(\sigma_i \cap \sigma_{i+1})}{\sqrt{\dim(\sigma_i)\dim(\sigma_{i+1})}} \right\rangle \approx 0.78$

3. 

4. Correct topology, $H_*(\mathcal{T}_{CA}) = H_*(\mathcal{T}) = H_*(\mathcal{E})$

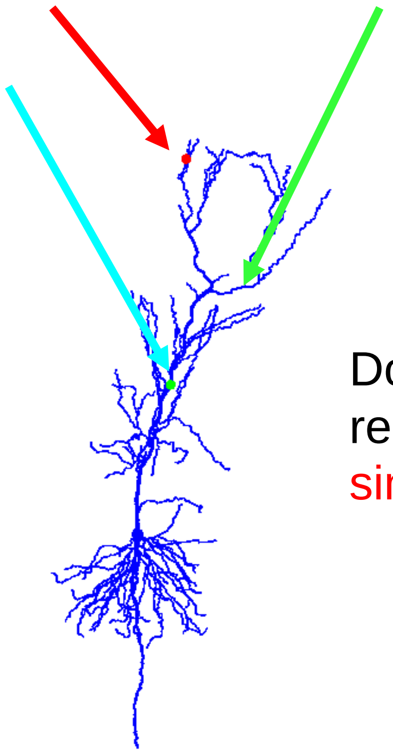
5. Learning times T_{min} are the approximately the same

Detecting place cell coactivity



Jarsky et al., *Nat. Neurosci* 2005

Detecting place cell coactivity, case 1



Downstream neurons
respond to (nearly)
simultaneous coactivity

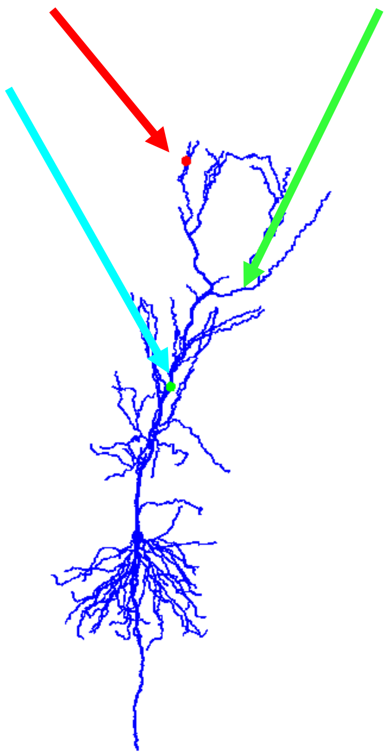
Jarsky et al., *Nat. Neurosci* 2005

Detecting place cell coactivity, case 2

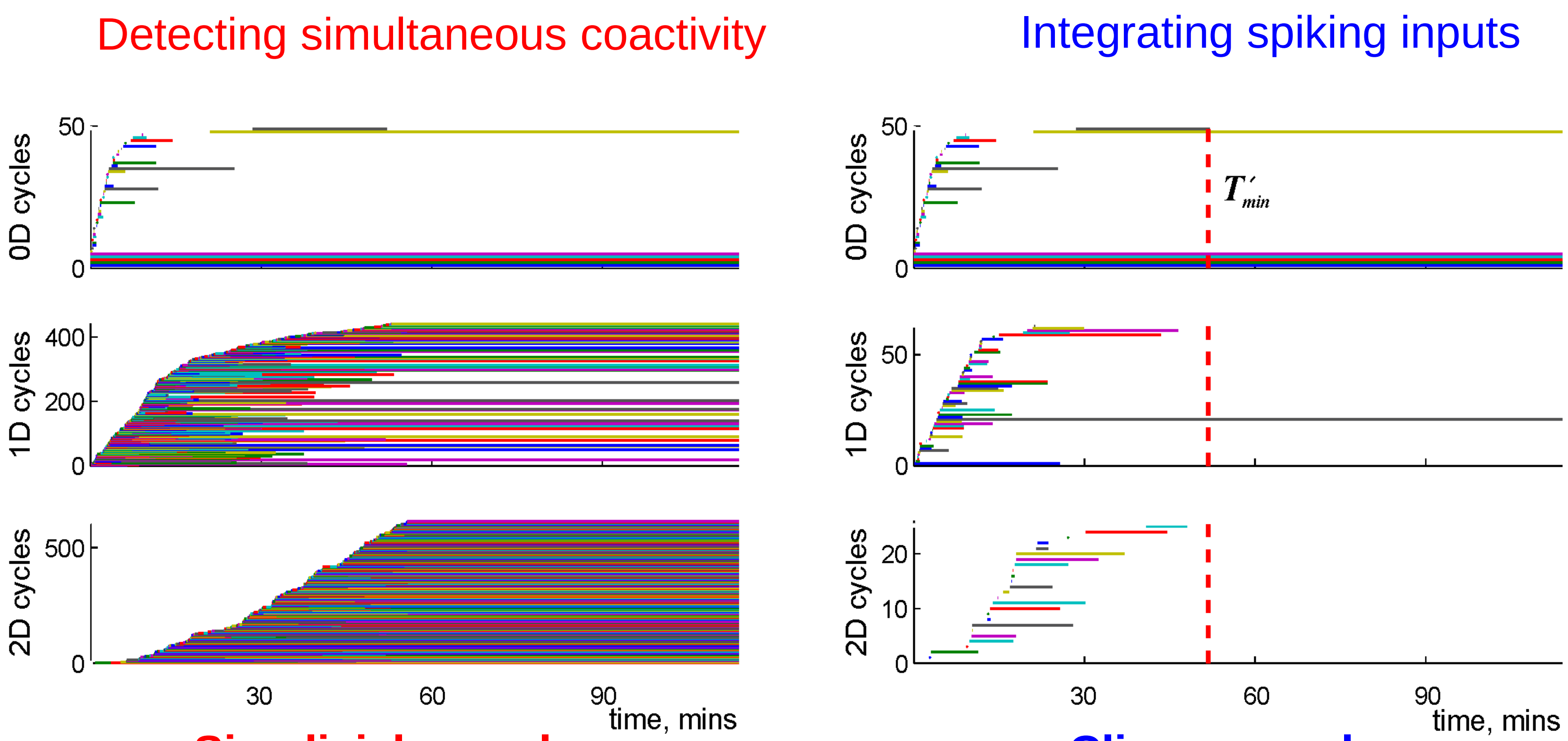
Downstream neurons
integrate inputs over
a time window ϖ

Jarsky et al., *Nat. Neurosci* 2005

Clique complex vs simplexes

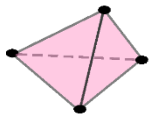


Jarsky et al., *Nat. Neurosci* 2005

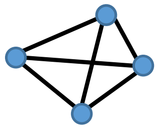


Simplicial complex

Clique complex



$\sigma_{ij\ldots k} = [c_i, c_j, \ldots, c_k]$

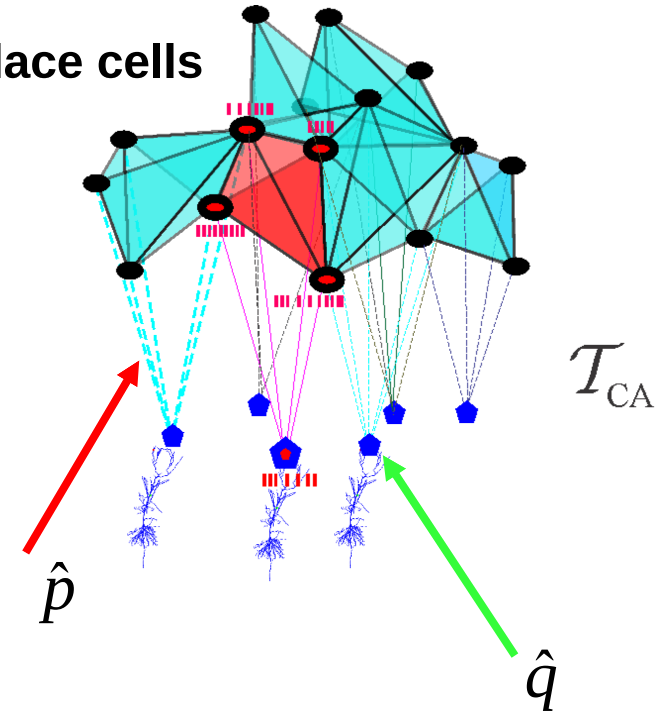


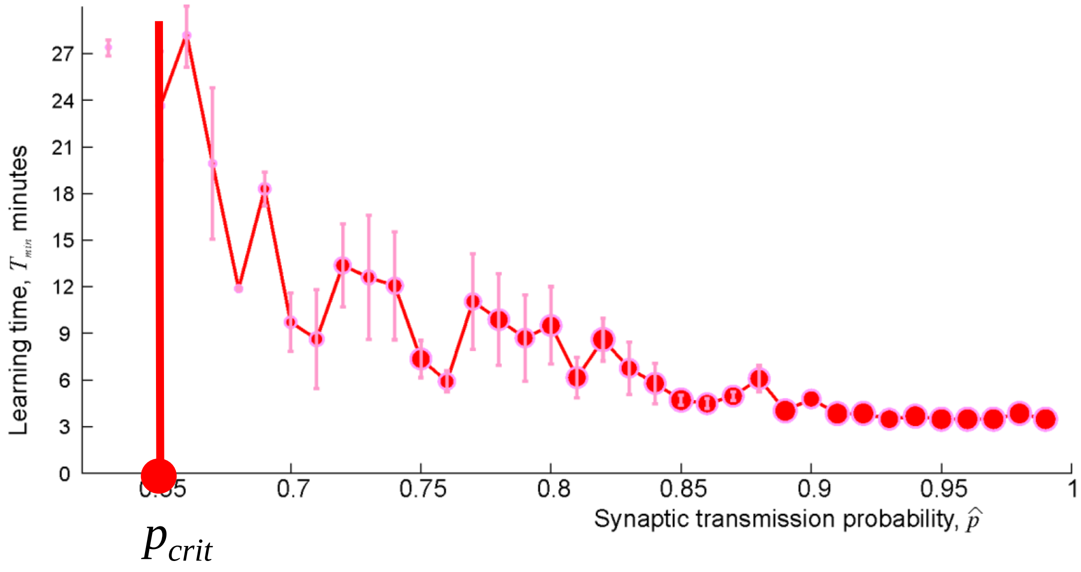
$\sigma_{ij\ldots k} = [c_i, c_j, \ldots, c_k]$

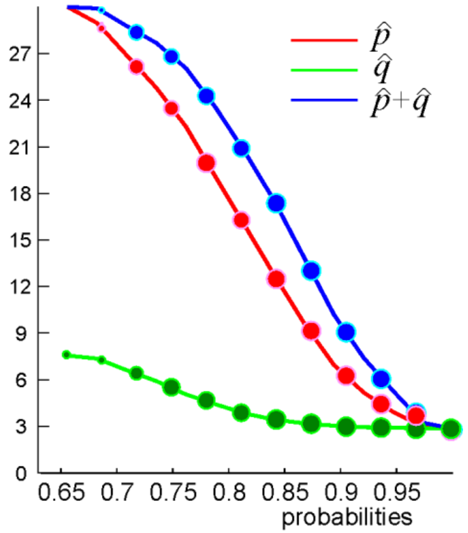
It is better to integrate inputs than to detect them. Especially for bats.

Effects of synaptic connectivity

Place cells

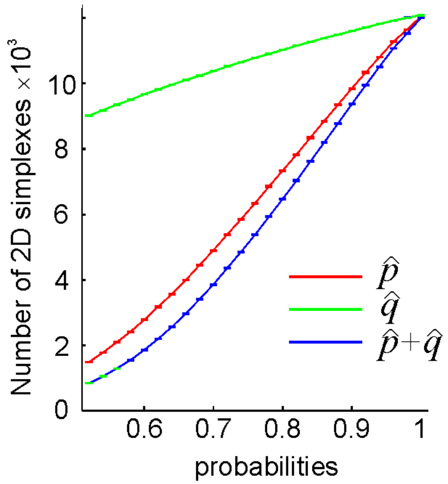


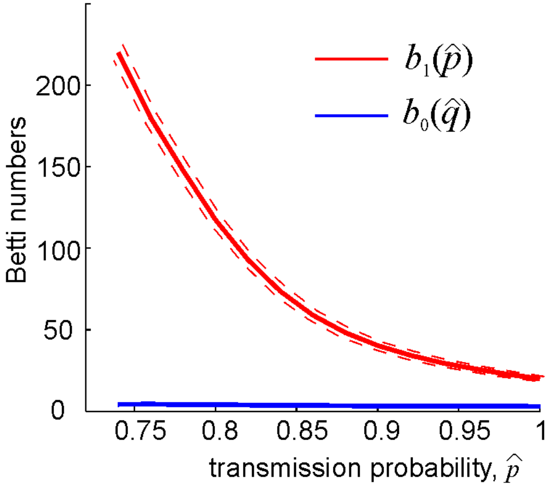


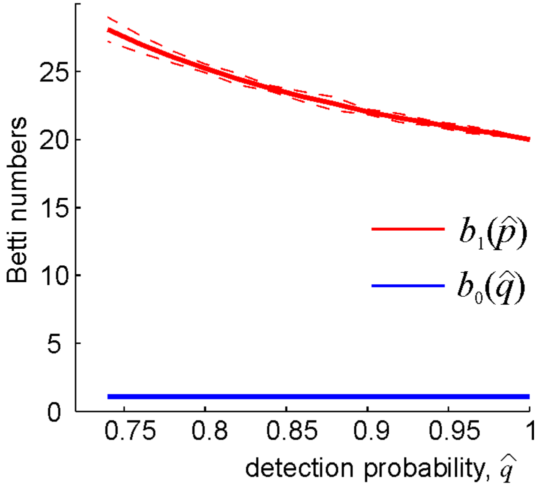


$$T_{\min} \propto \left(\hat{p} - \hat{p}_{crit}\right)^{-k}$$

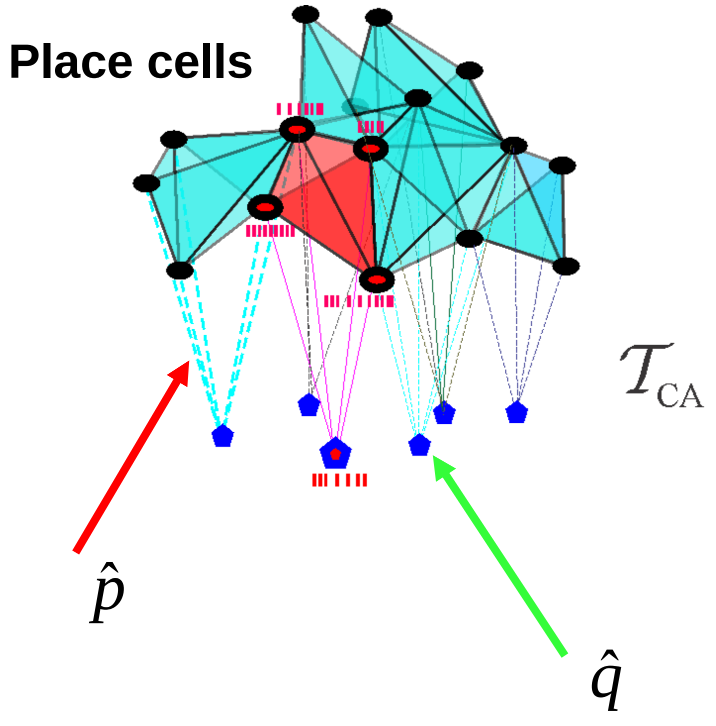
$$T_{\min} \propto \left(\hat{q} - \hat{q}_{crit}\right)^{-\kappa}$$



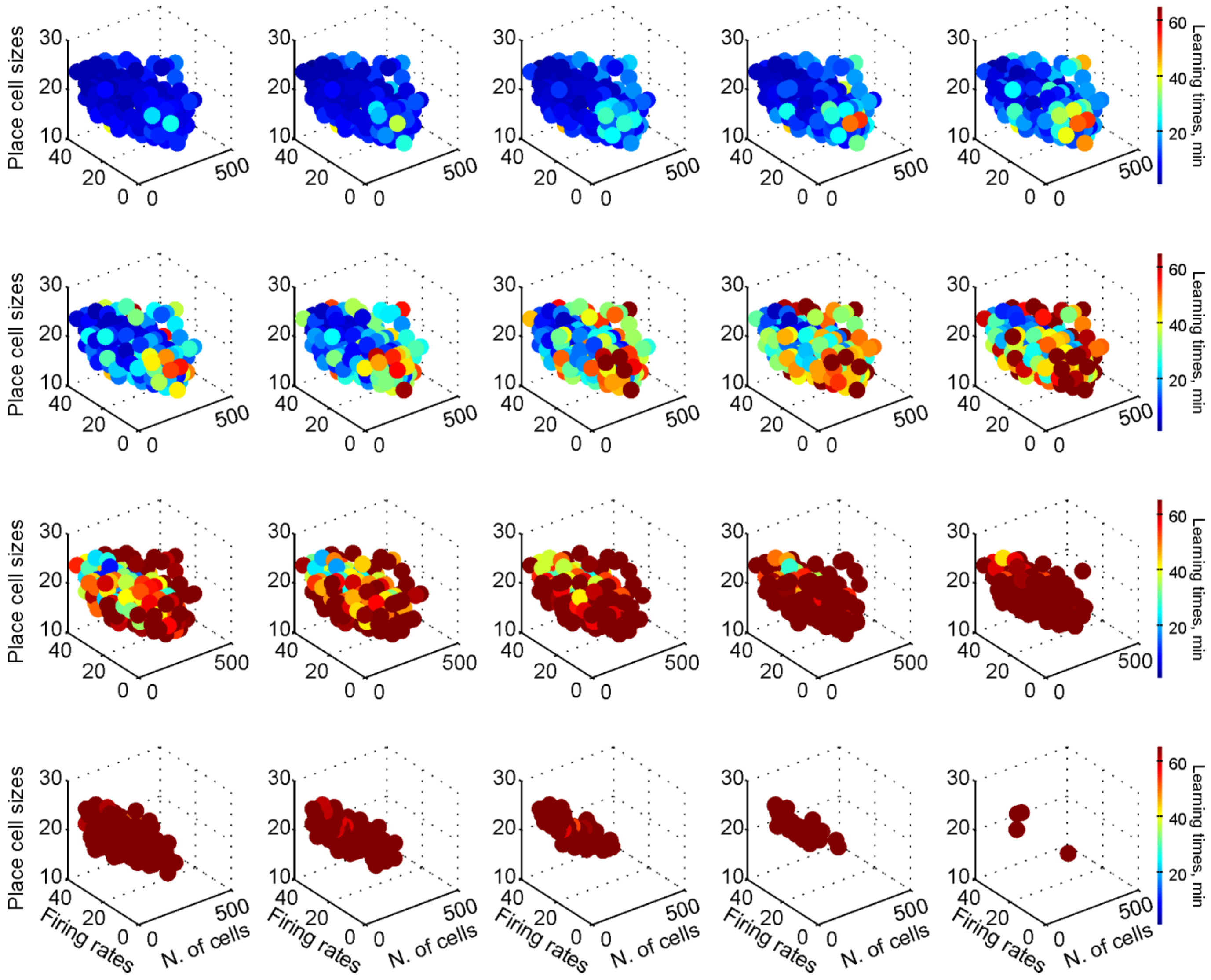




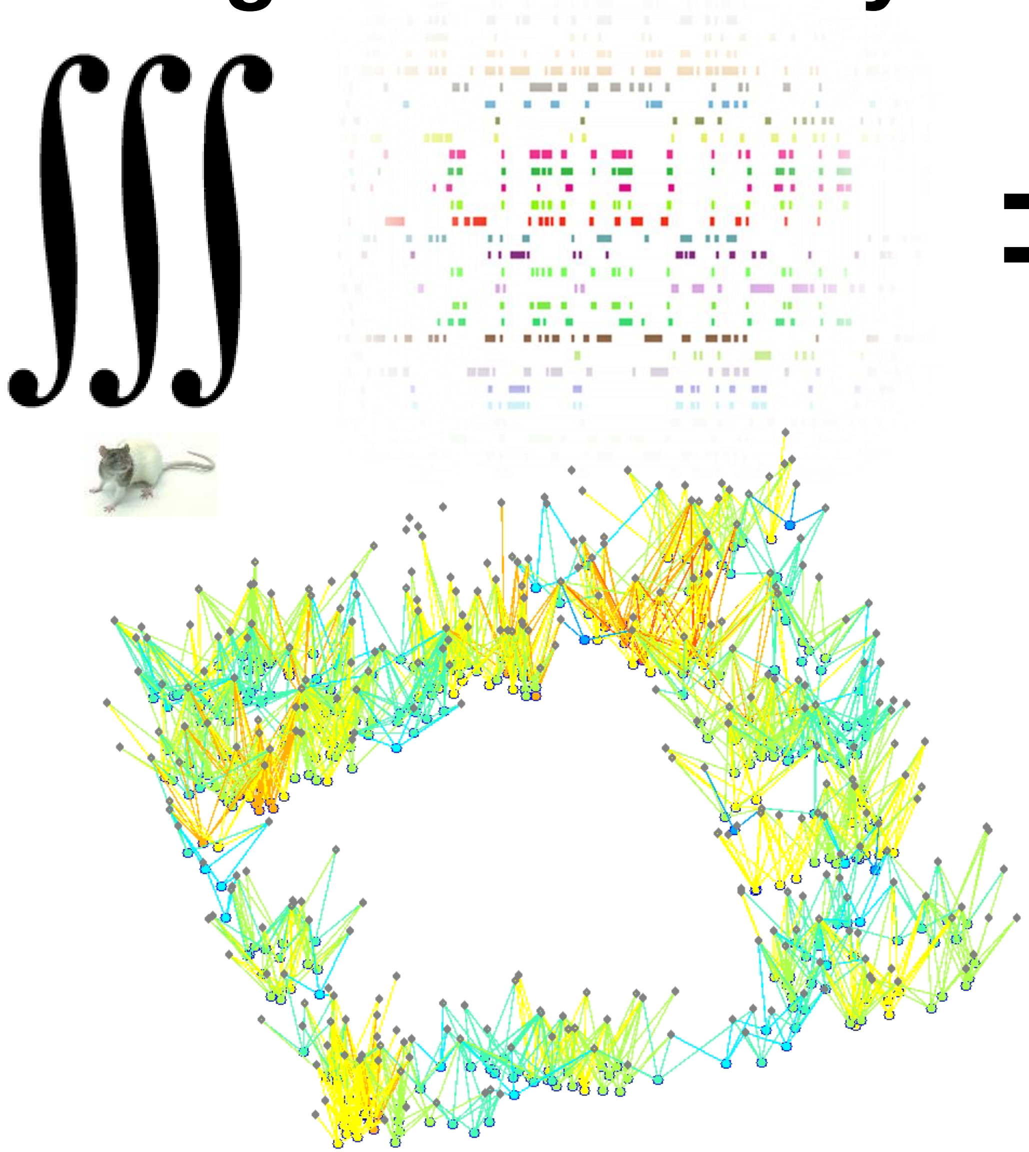
Effects of synaptic connectivity



$$T_{\min} \propto \left(\hat{p} - \hat{p}_{crit}\right)^{-k}$$



Rewiring cell assembly network



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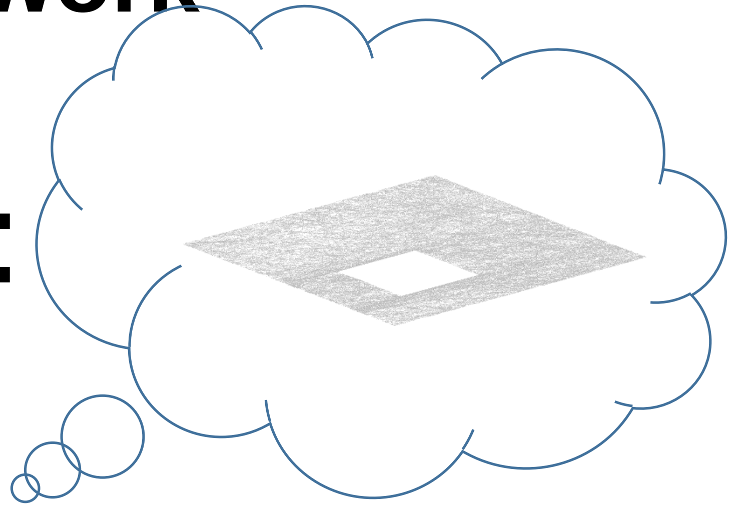
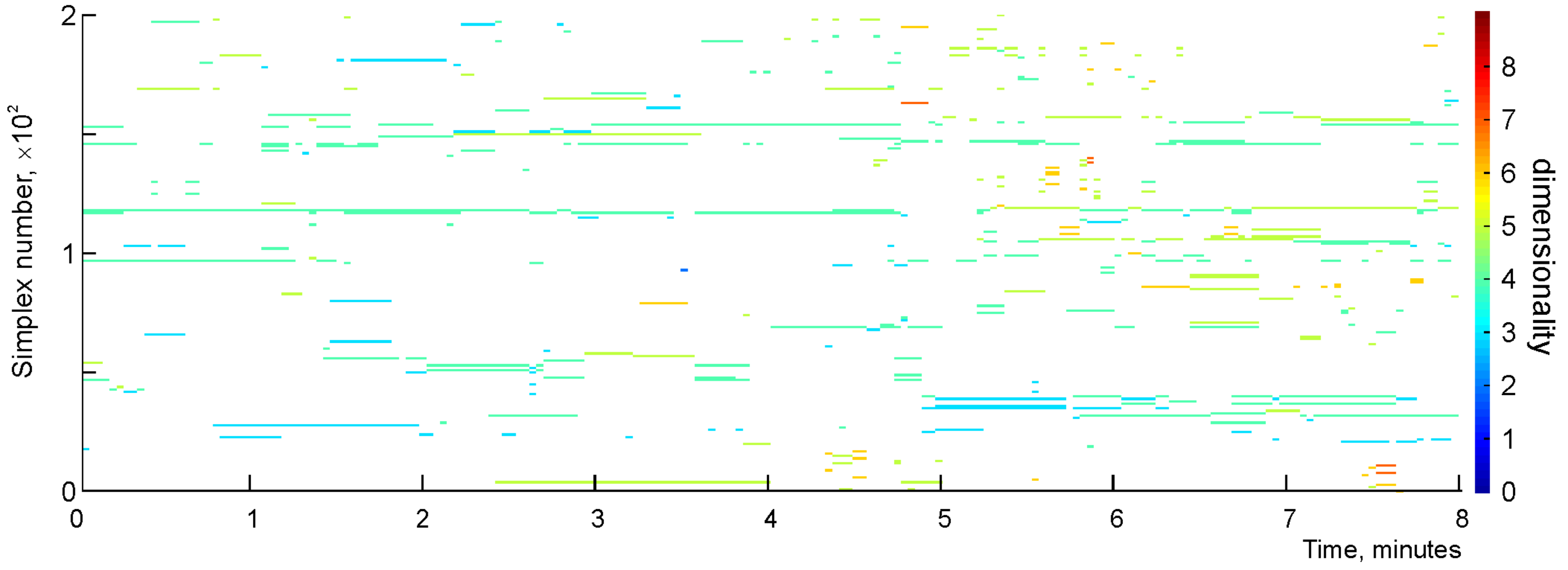
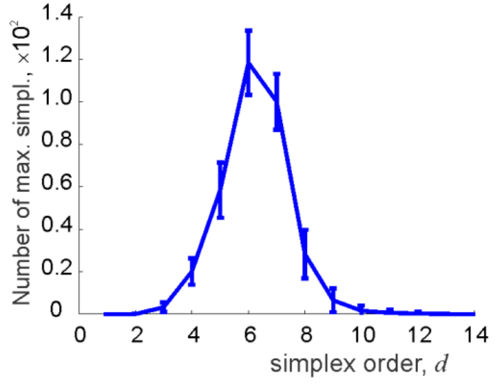


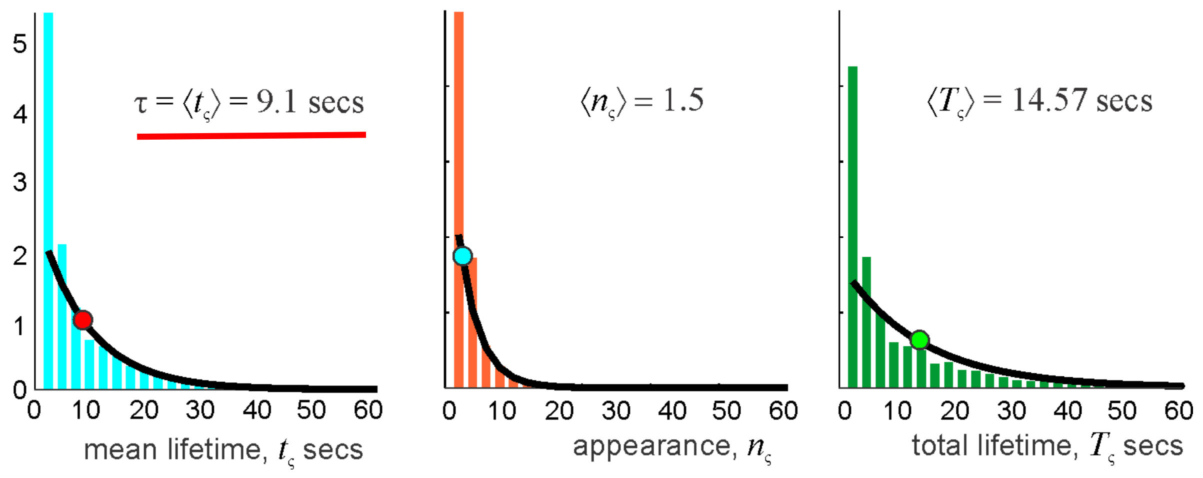
Cell assemblies become unstable
Cognitive maps remain stable

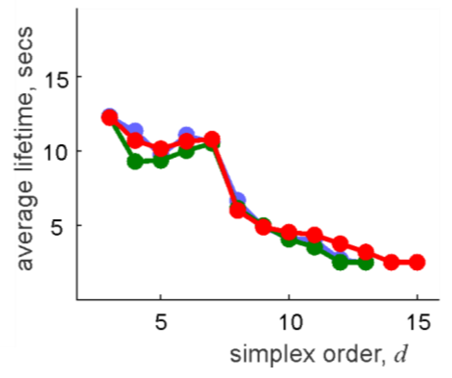
How can that work?

Transient (finite time) cell assemblies

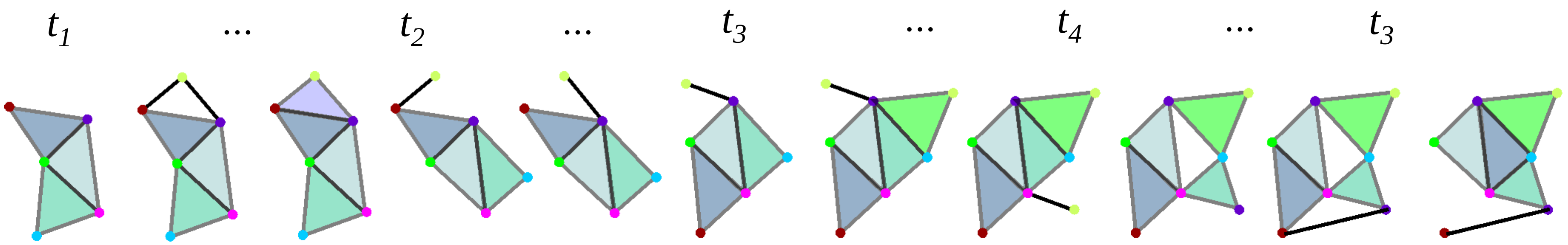






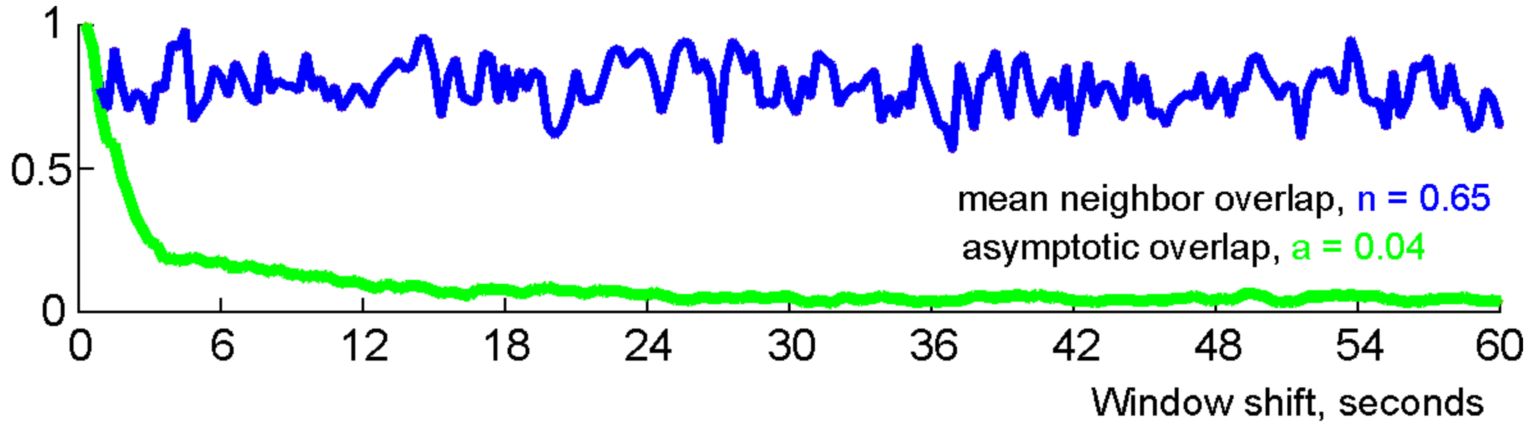


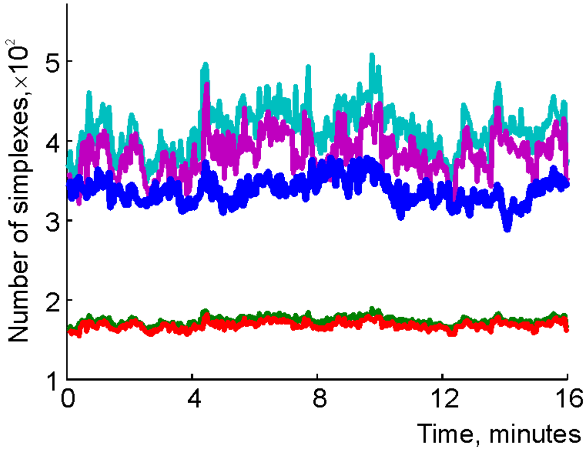
Simulations of flickering network



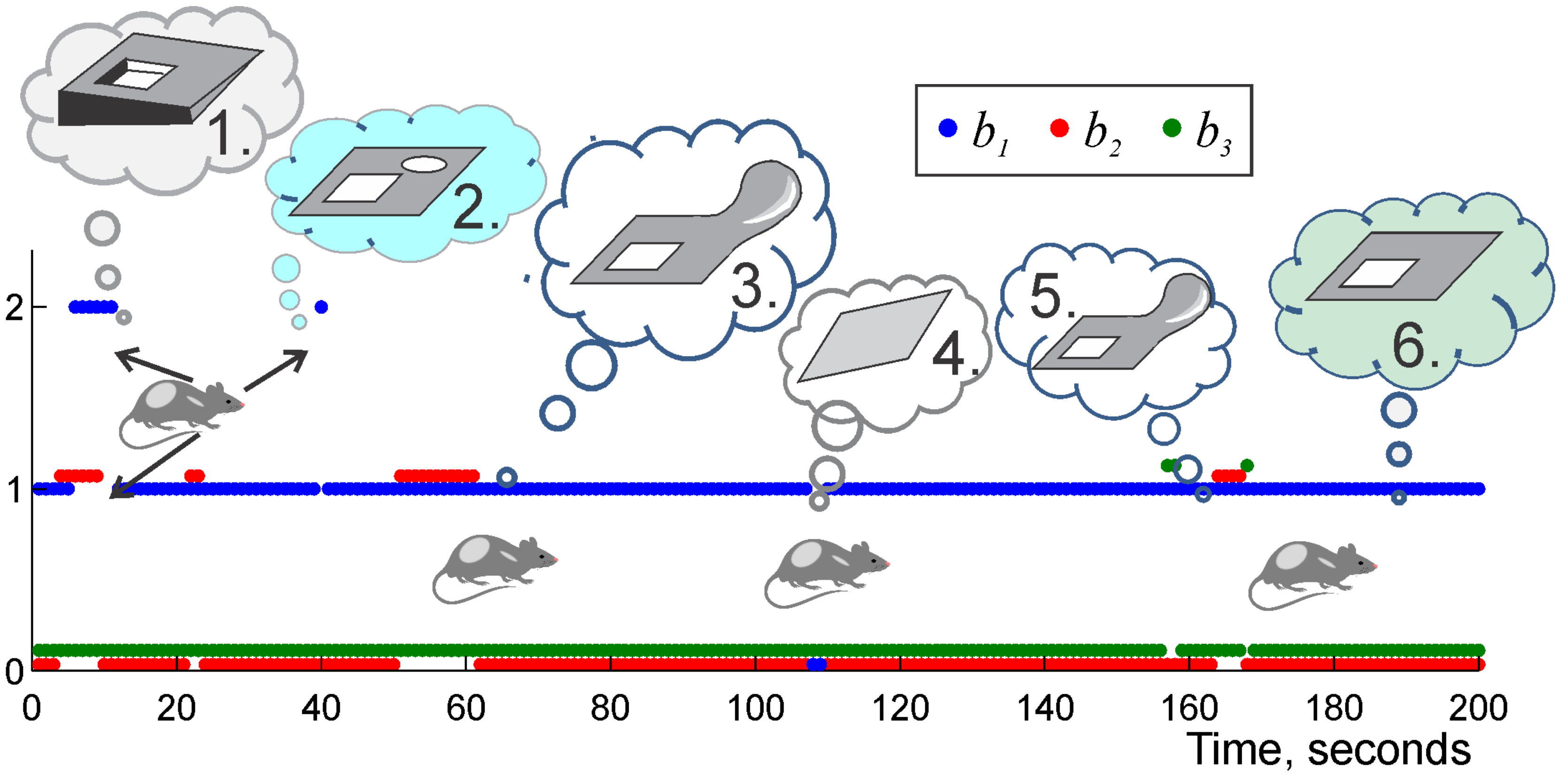
$\langle \mathcal{F}_\tau(t)\mathcal{F}_{\tau+1} \rangle$

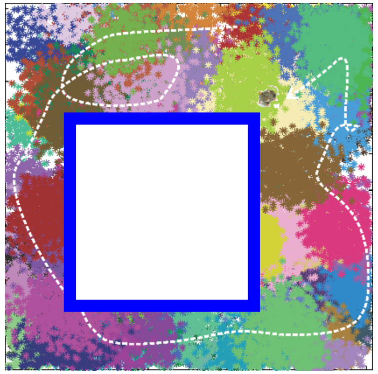
$\langle \mathcal{F}_\tau(t)\mathcal{F}_{\tau+t} \rangle$





Rewiring cell assembly network encodes a stable map



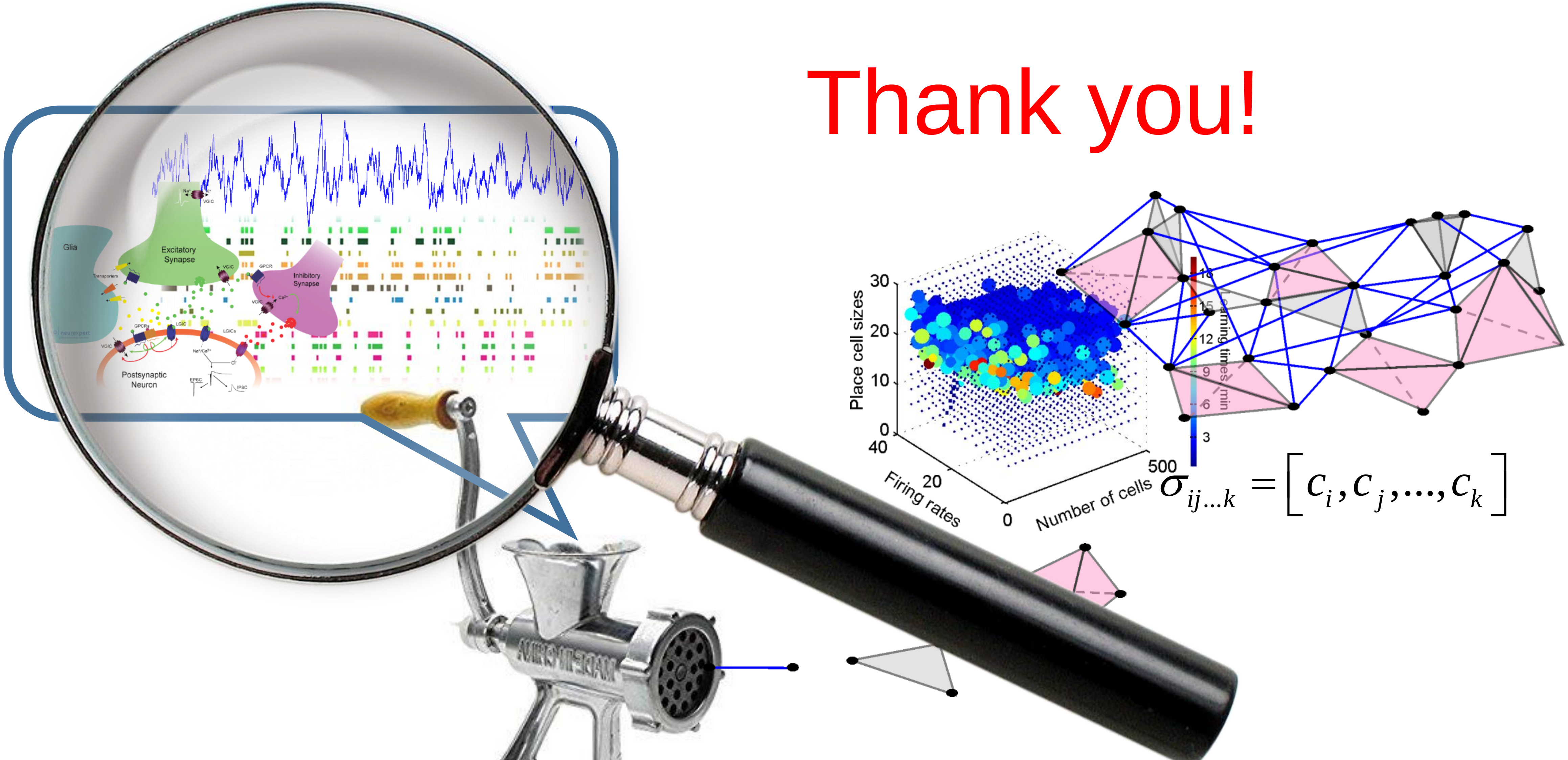


Number of holes

Number of bubbles

Number of 3D bubbles

Thank you!



- NSF grant 1422438
- University of Texas Startup Funds