

$$\int_{-\infty}^{\infty} e^{i\pi x^2} dx = \sqrt{\frac{1}{i}} \lim_{\epsilon \rightarrow 1^-} \sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} a_n$$

$$\frac{\partial_t u + u \partial_x u + \partial_x^3 u = 0}{\varphi(s+t) = \frac{\varphi(s)\sqrt{1-\varphi(t)^4} + \varphi(t)\sqrt{1-\varphi(s)^4}}{1+\varphi(s)^2\varphi(t)^2}}$$

$$\int_0^1 \frac{1}{\sqrt{1-z^4}} dz = \frac{1}{2} \int_0^1 \frac{1}{\sqrt{1-p^{-s}}} dp$$

$$u_t - u_{xxt} + 3uu_x - 2u_x u_{xx} - uu_{xxx} = 0$$

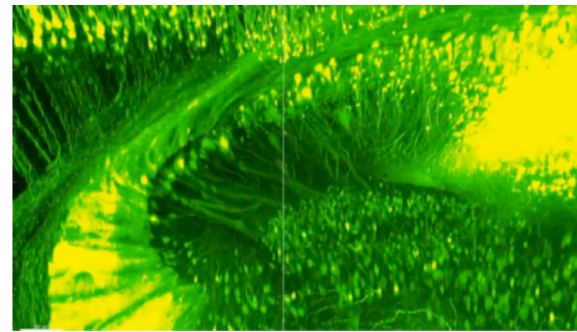
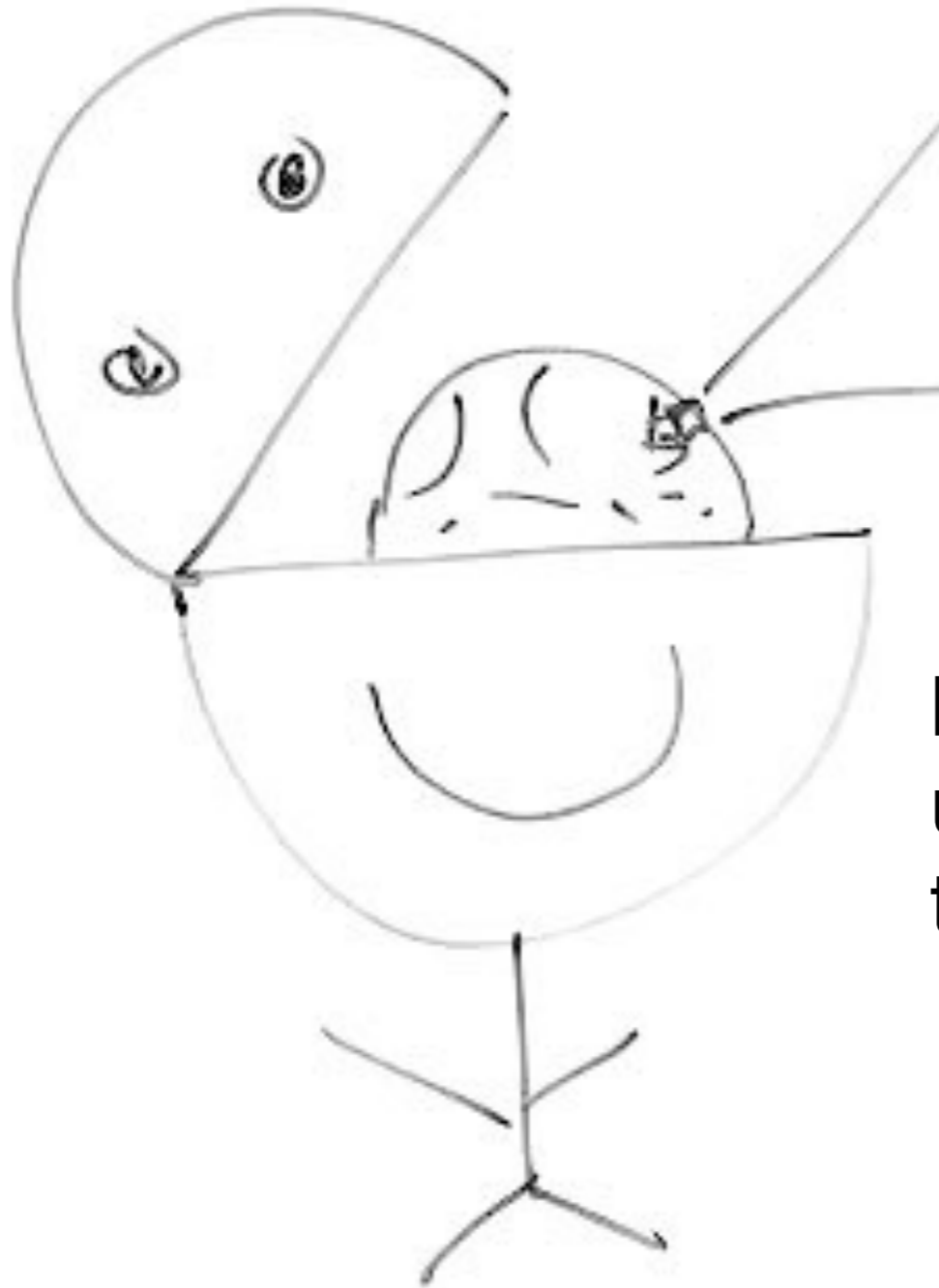
Department of Mathematical Sciences



Shape of neural state space

Work largely based on:
Gard Spreemann et al., 2015/2018
Erik Rybakken et al., 2017
 And **Nils Baas**

Neuroscience



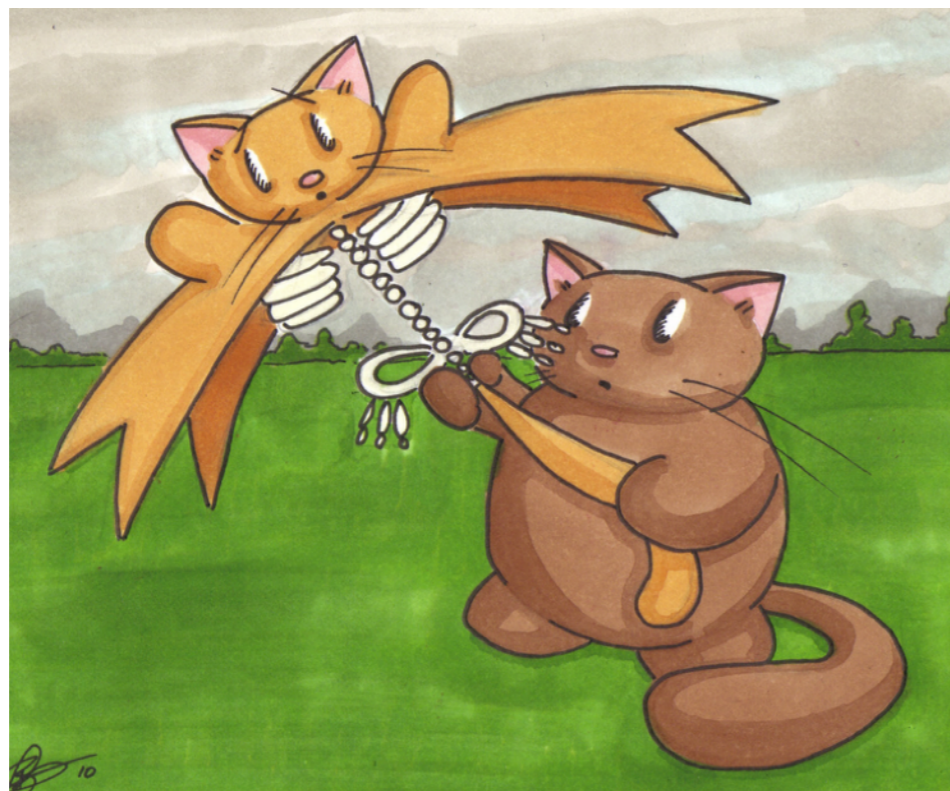
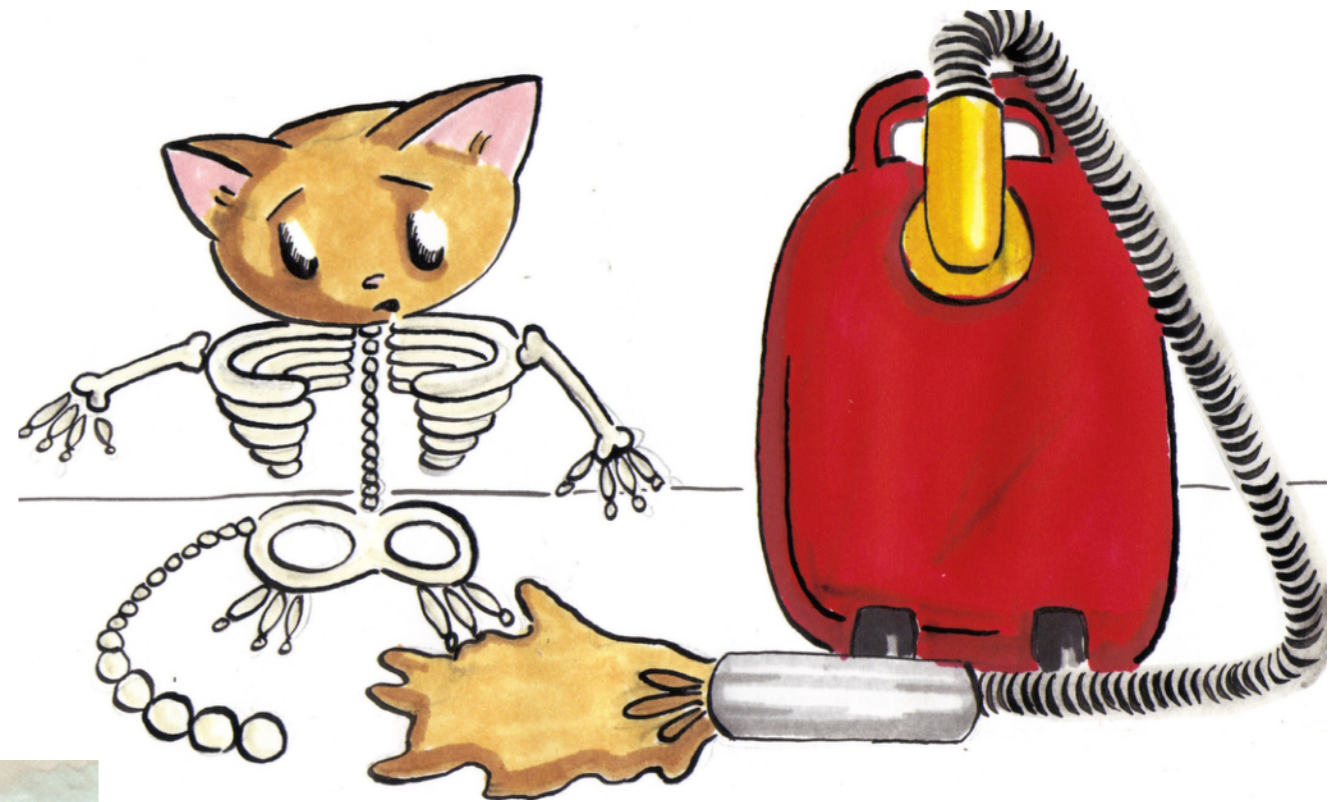
Cheung et al., 2013



how do these enable
us to do interesting
things?

How to figure out how the brain works:

- Perturb it
- Take it apart
- **Observe it**
- Model it
- etc.

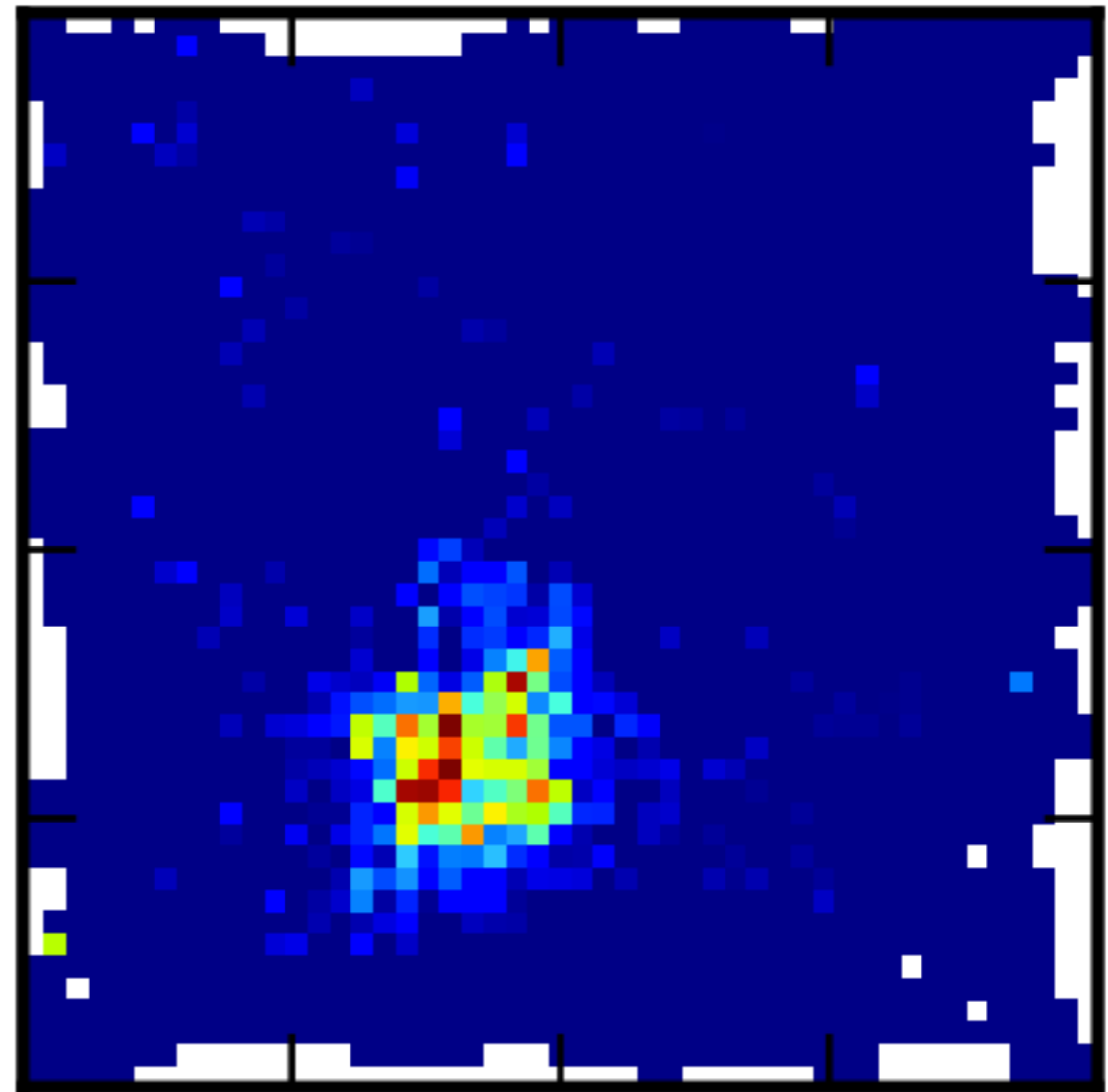
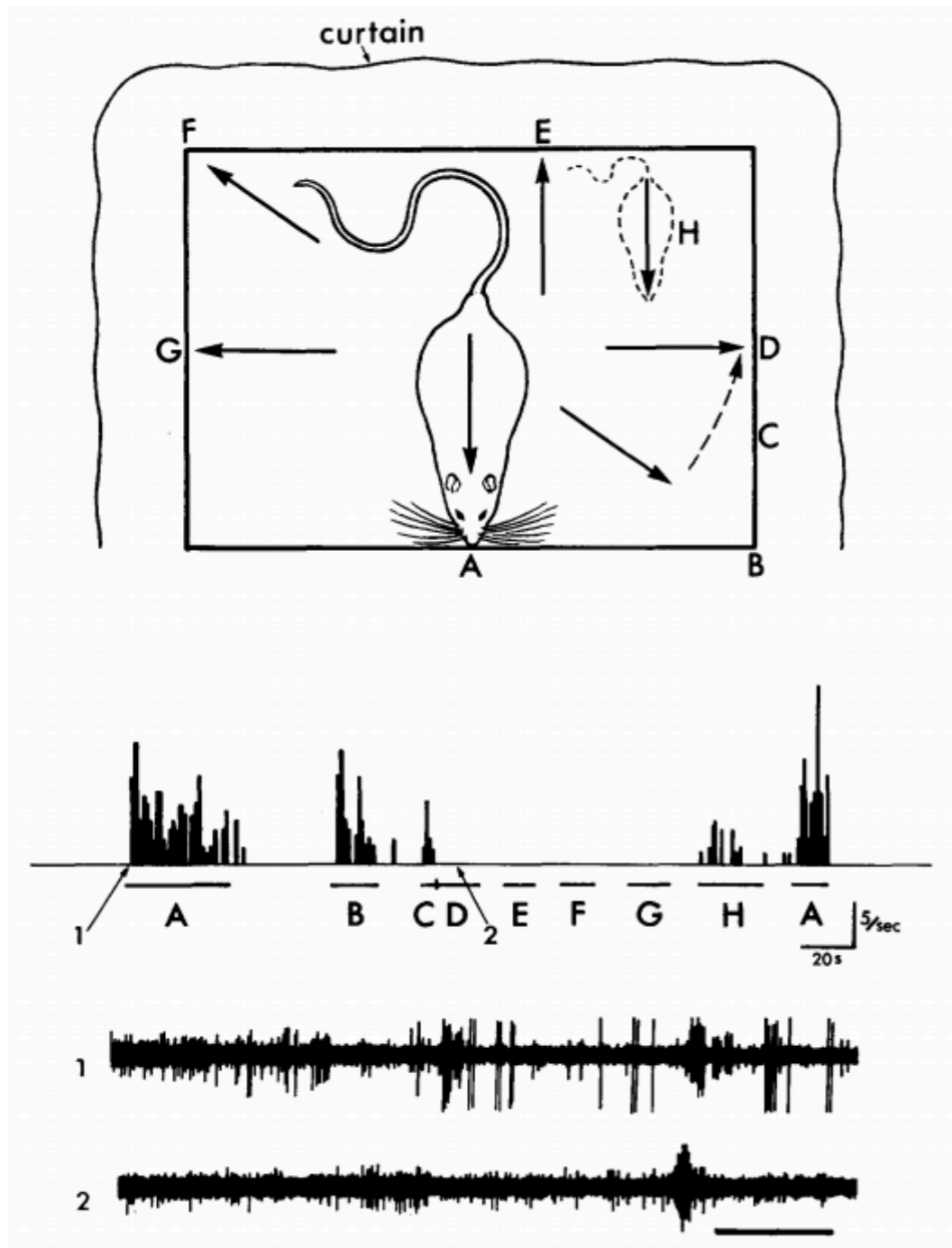


Example - place cell

“Unit activity was monitored during spontaneous behaviours such as **walking, eating, drinking, grooming, and sleeping**, and during elicited behaviours such as **orienting, sniffing at cotton wool or various odours, biting at a polyethylene tube, and in some instances, bar-pressing for food**. Responses to simple **auditory (clicks, whistles, scratching noises), visual (moving light, hand, black and white striped board), olfactory (various odours, rat feces), and tactile (touch and pressure over the body surface) stimuli were also tested...** This preliminary paper will concentrate on the response properties of **8 of the 76 units** obtained in 36 electrode penetrations through the dorsal hippocampus (fields CA1 and 4) and the dentate gyrus in 23 rats. Of the remaining 68 units, **14 were classified as 'arousal' or 'attentional' units** and resemble those reported by Vinogradova et al.; **21 'movement' units** had patterns of activity directly related, to the animal's behaviour, firing briskly during some but not necessarily all of the following behaviours: orienting, sniffing, bar-pressing, and walking, and firing infrequently or not at all, during eating, drinking, grooming, quiet sitting, and slow wave sleep*; **two units** had interesting properties relating to the **animal's expectations**; and for the remaining **31, either no adequate stimulus or behaviour could be found** or their responses were inconsistent and uninterpretable. This last group includes **15 units which**, apart from an occasional spike in conjunction with a burst of spikes in several smaller units, **remained silent in spite of considerable, and sometimes drastic, attempts to fire them.**”

O'Keefe and Dostrovsky, 1971

Example - place cell



O'Keefe and Dostrovsky, 1971

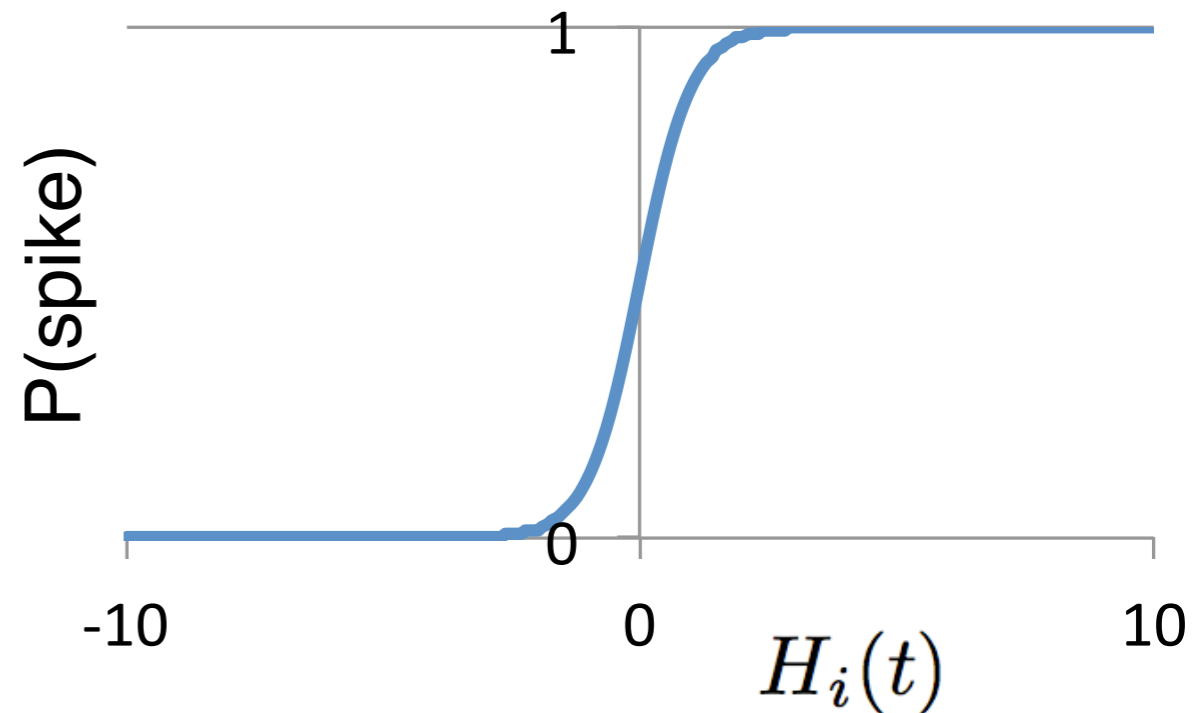
Model selection with GLMs

$$P(S_i(t+1) | \mathbf{S}(t)) = \frac{\exp[S_i(t+1)H_i(t)]}{2 \cosh H_i(t)}$$

$H(t)$ can include many things:

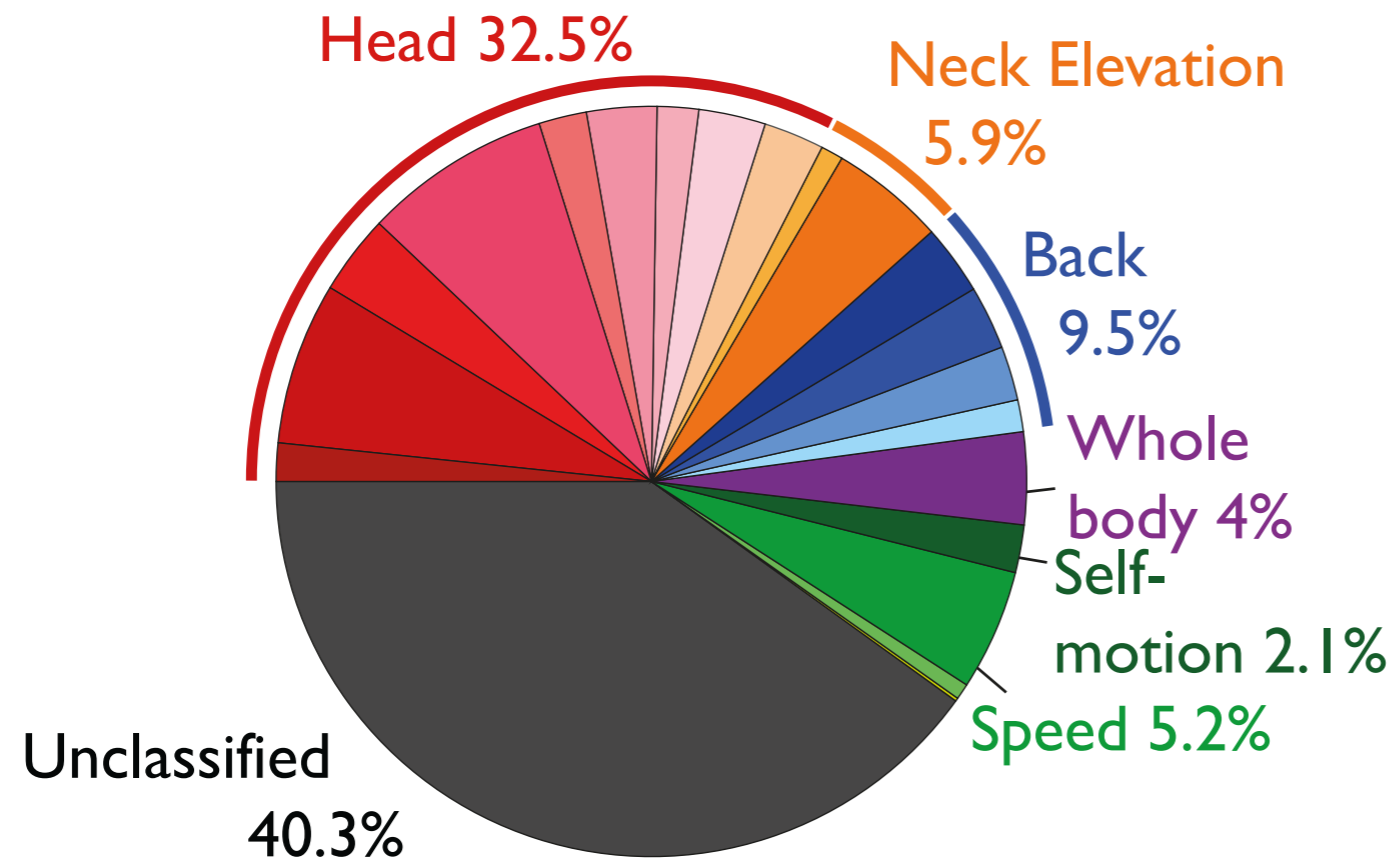
- input from other cells
- cues in the environment
- aspects of the experiment (e.g. head direction)
- motor things etc.

$$H_i(t) = h_i + \sum_j J_{ij} S_j(t)$$



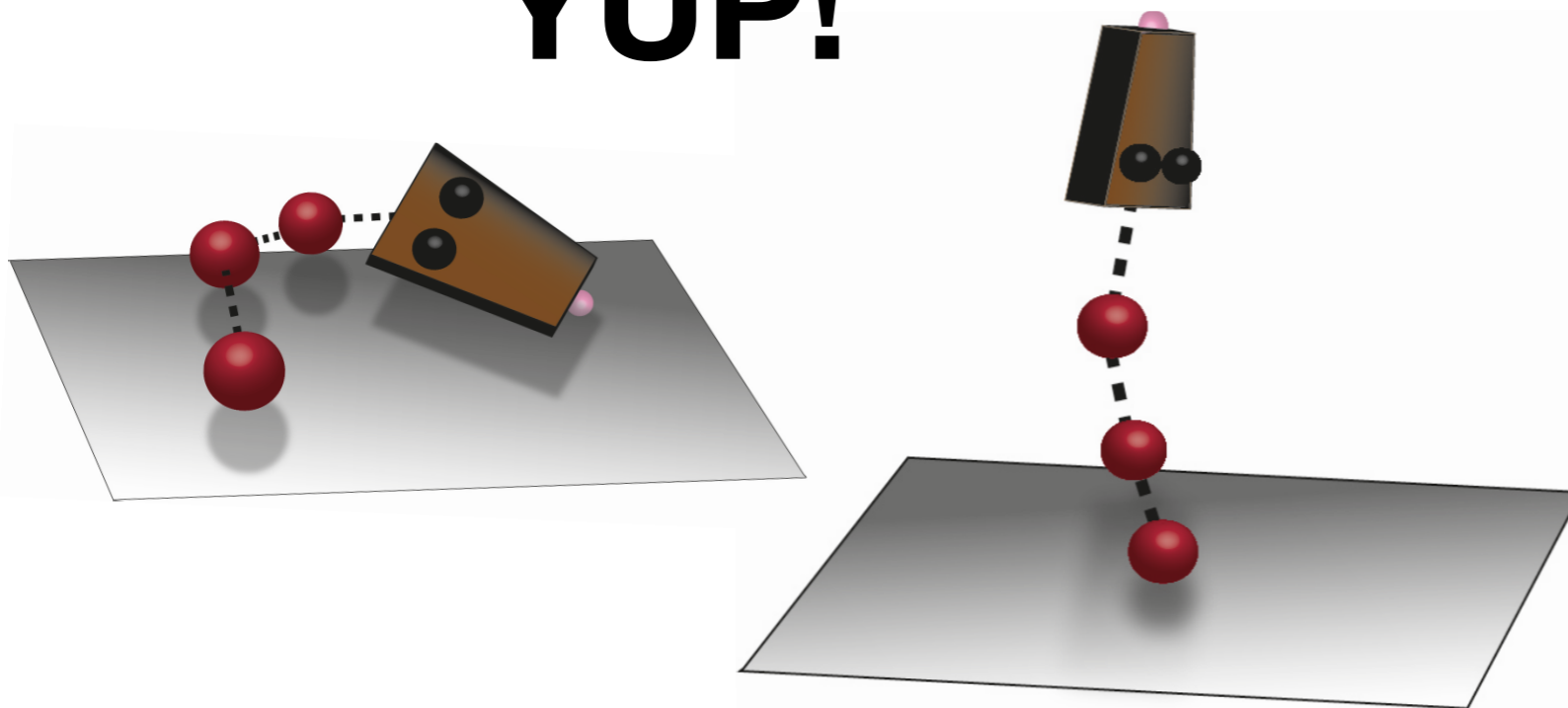
Truccolo et al., 2005,
Nelder & Wedderburn, 1972,
McCullagh & Nelder, 1989,
Roudi et al., 2015

Model selection: tedious but important



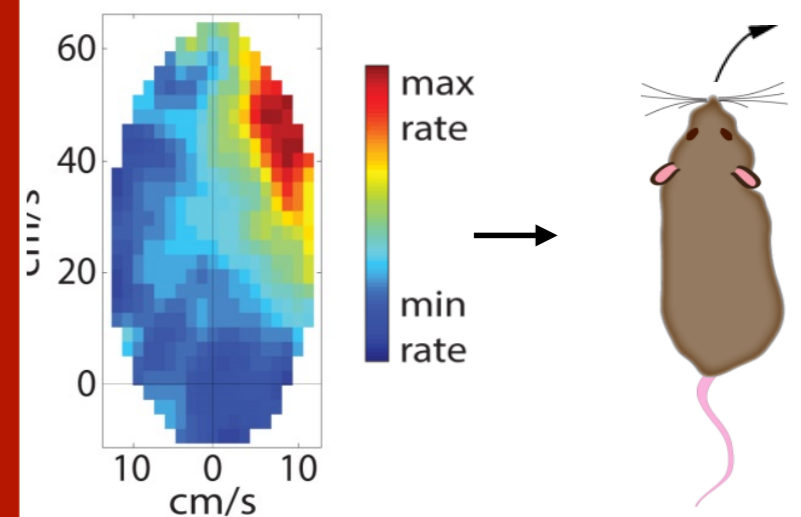
Parietal cortex

YUP!



rate map

NOPE

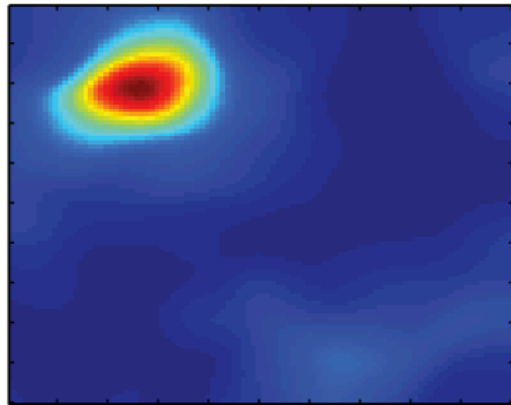


Mimica et al., 2018

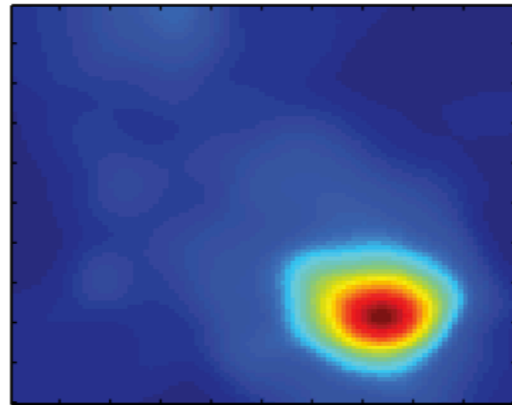
This typical way of doing model selection can be:

- Boring
- Time consuming
- Trying everything is impossible
- Easily “tampered”

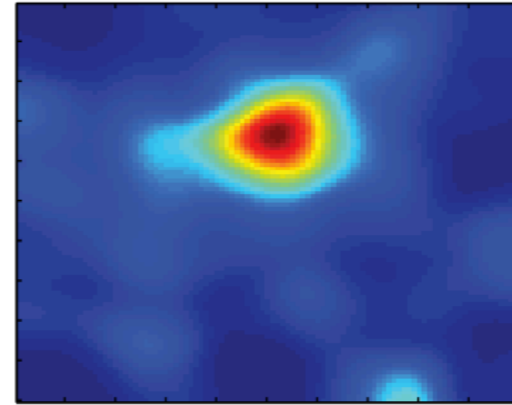
place field of neuron #1



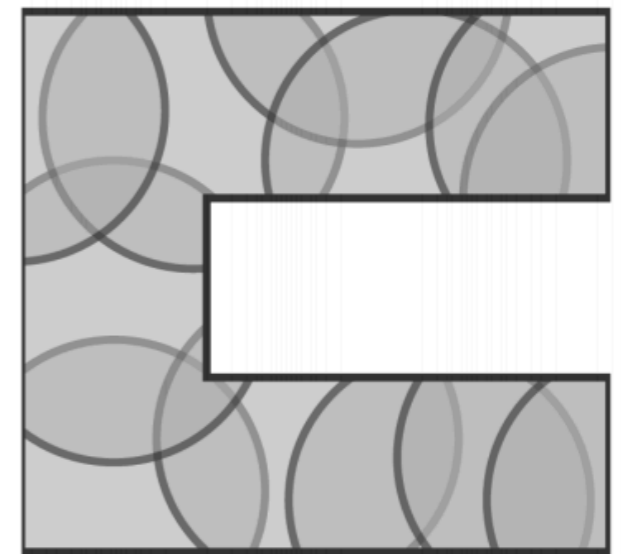
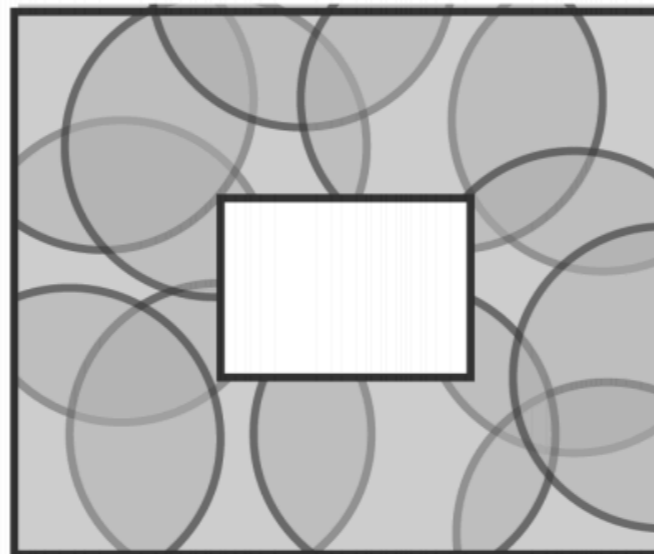
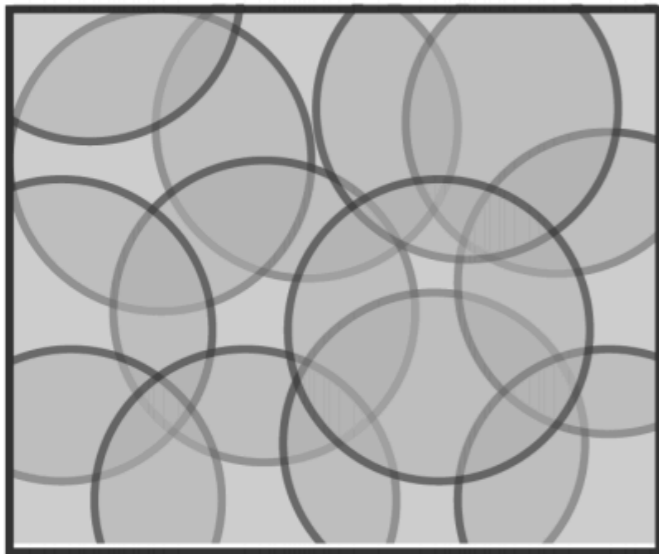
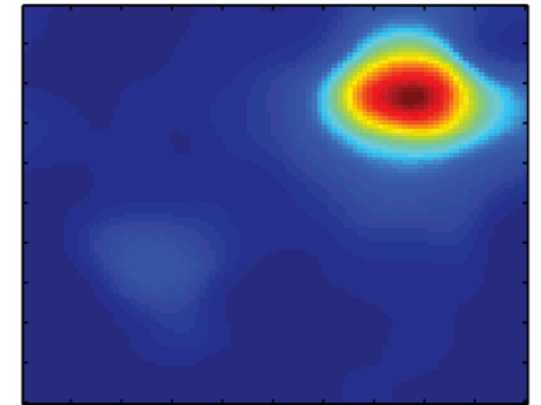
place field of neuron #2



place field of neuron #3

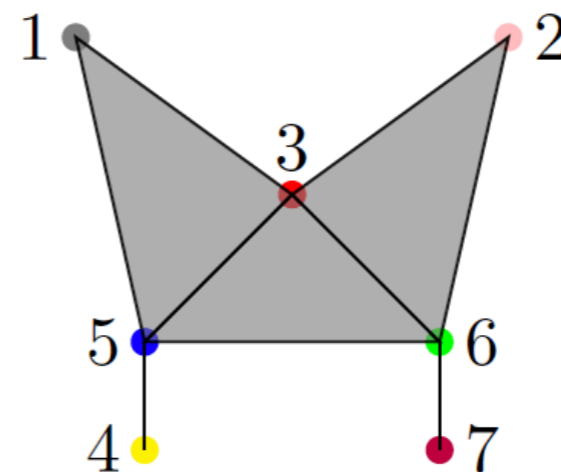
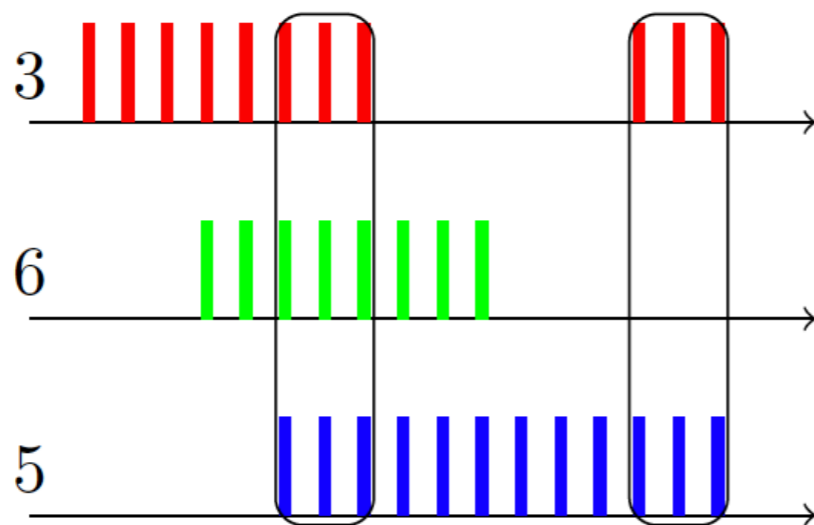
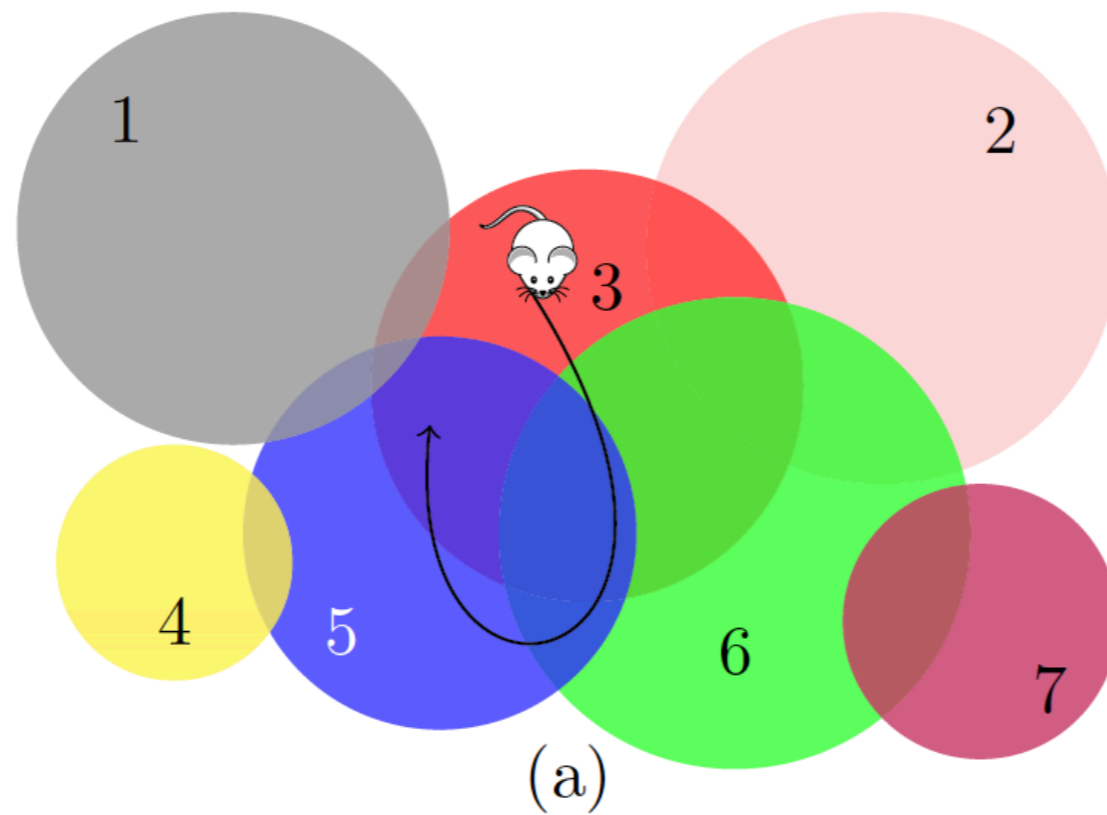


place field of neuron #4



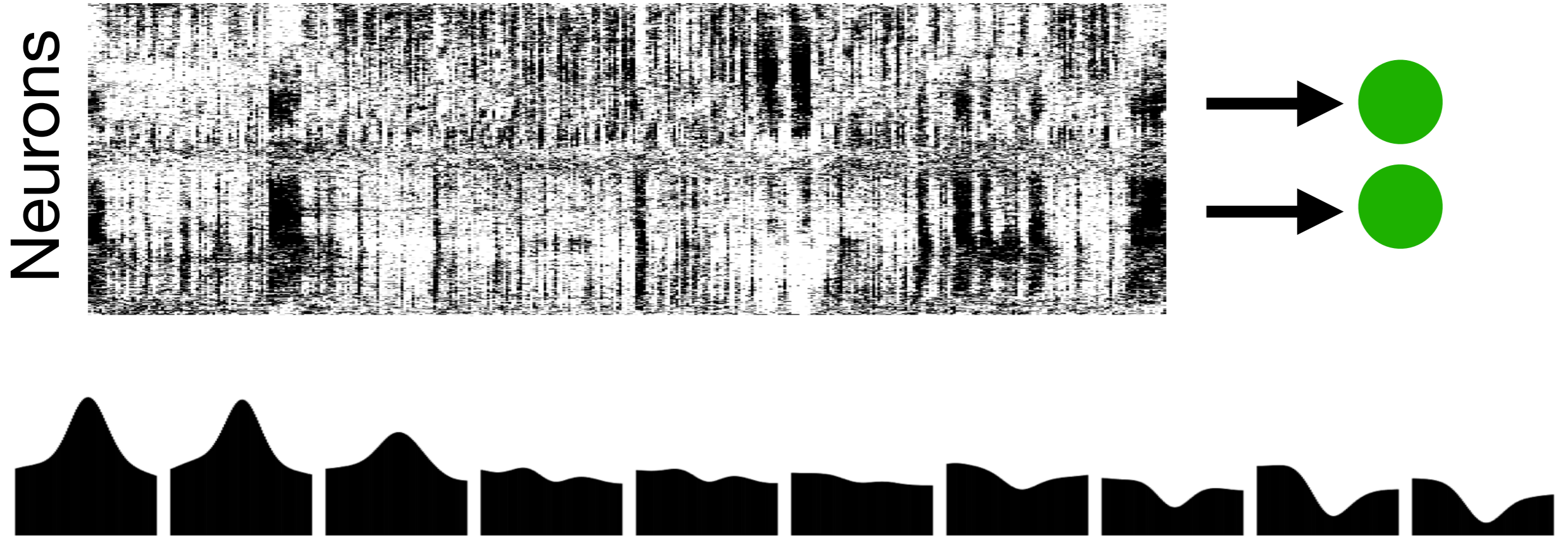
Singh et al., 2008
Curto and Itskov, 2008
Curto, 2016

From data to space



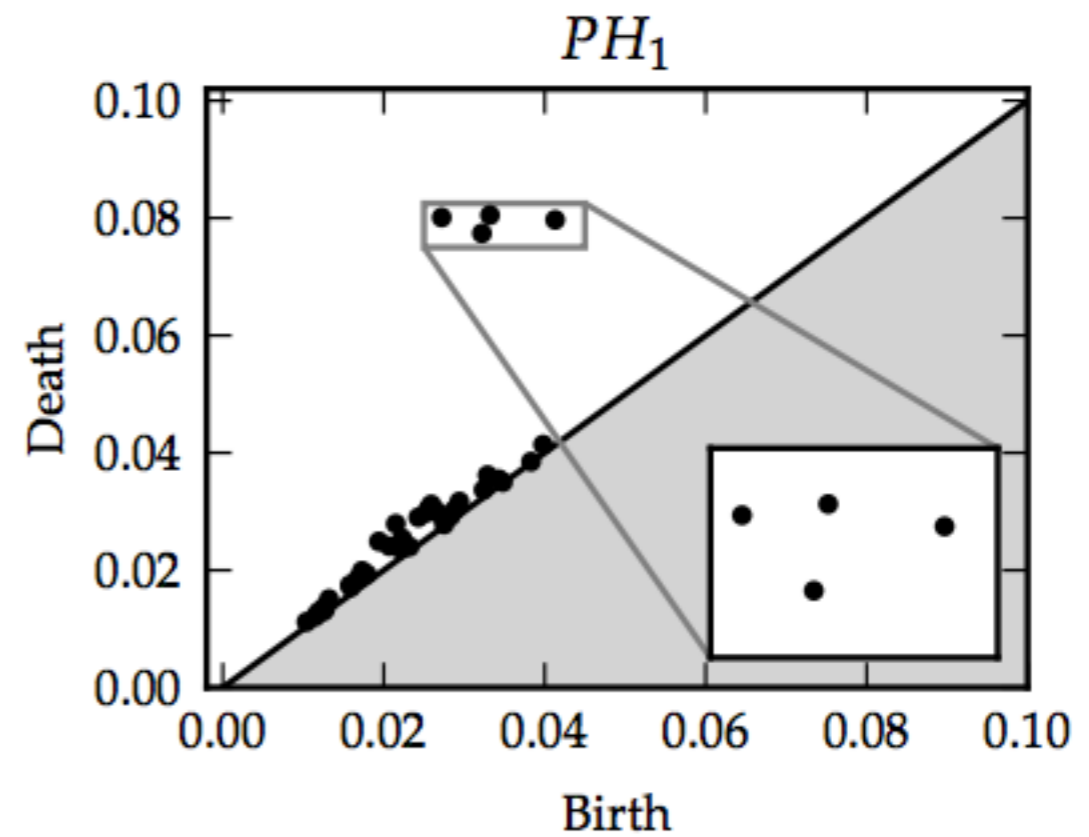
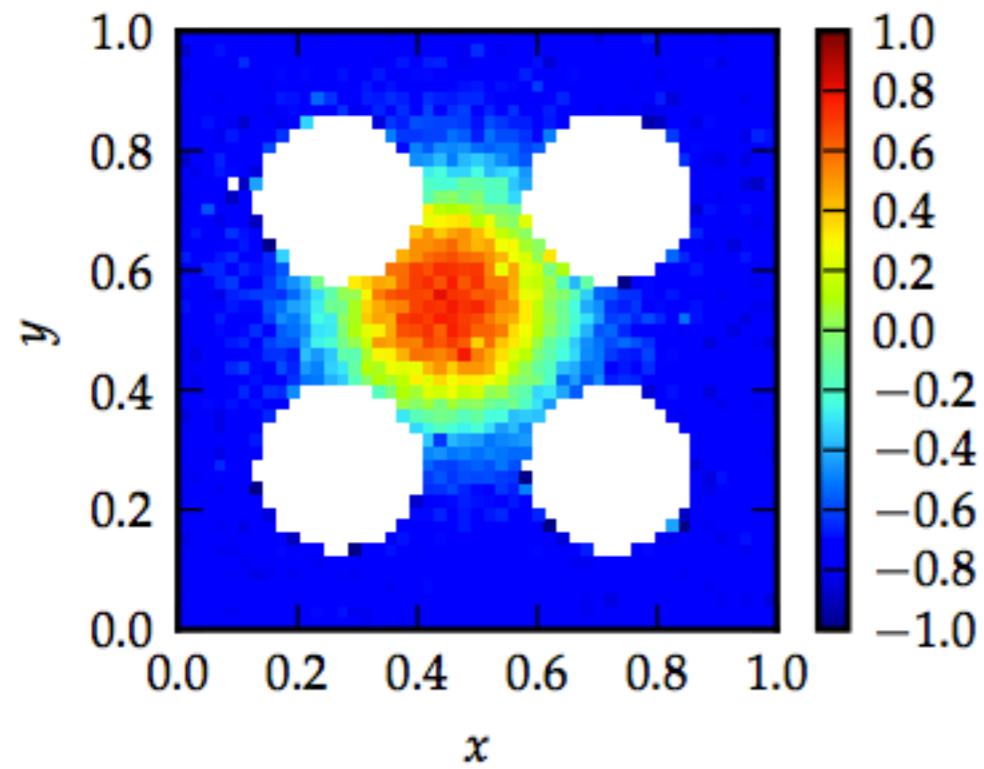
Singh et al., 2008
Curto and Itskov, 2008
Spreeman et al., 2015/18

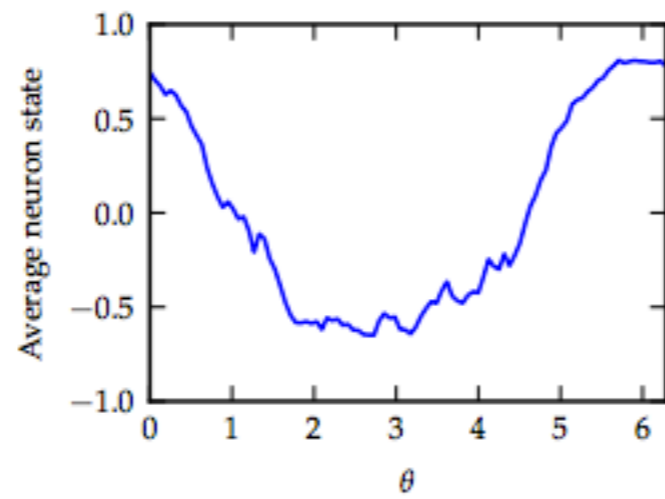
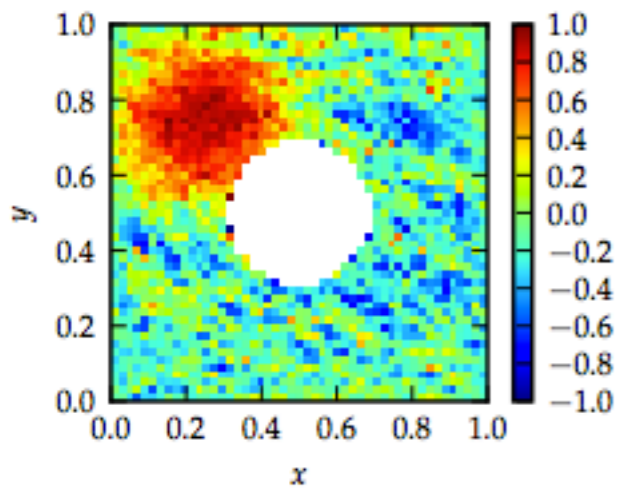
Neurons as vertices



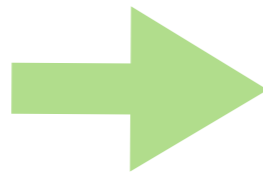
1 - Pearson correlation seemed reasonable

Example with four holes

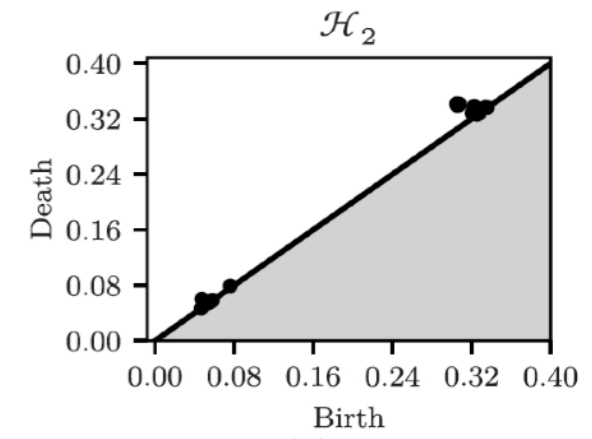
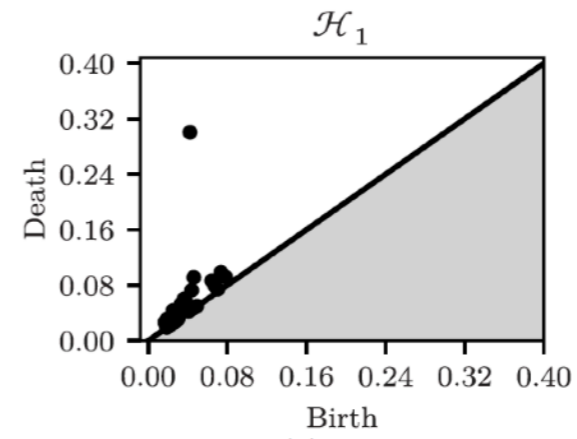
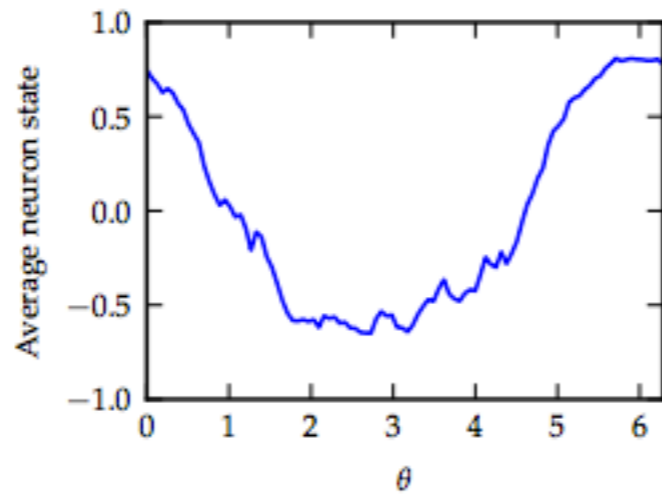
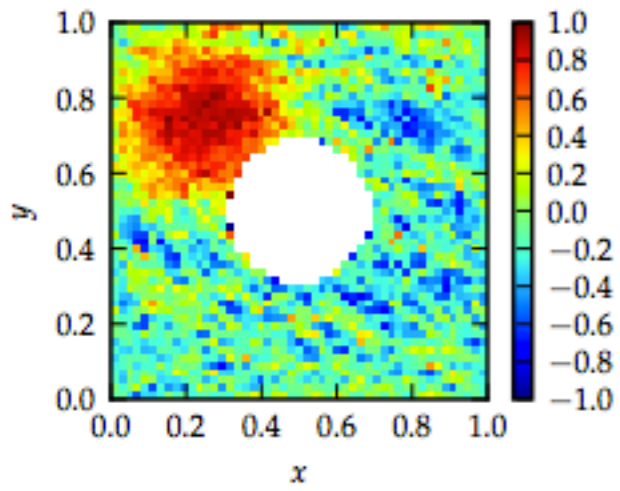




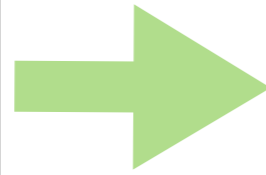
recording
device



START



recording device

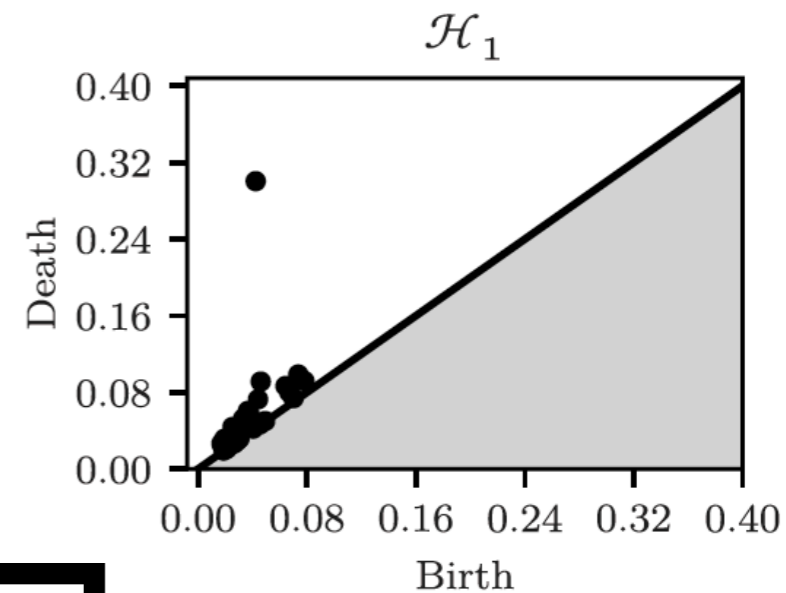
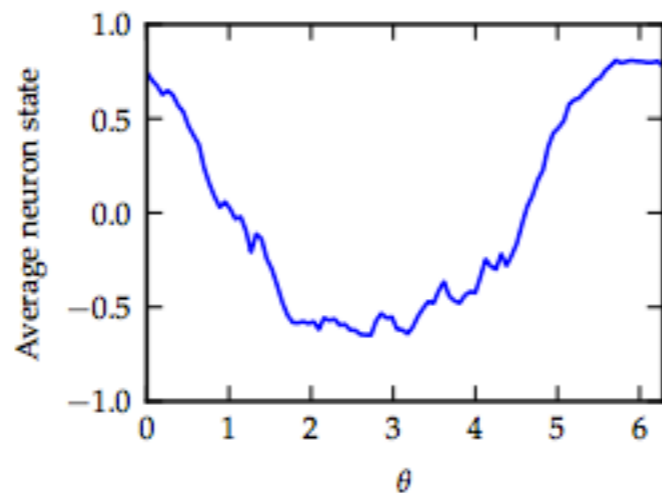
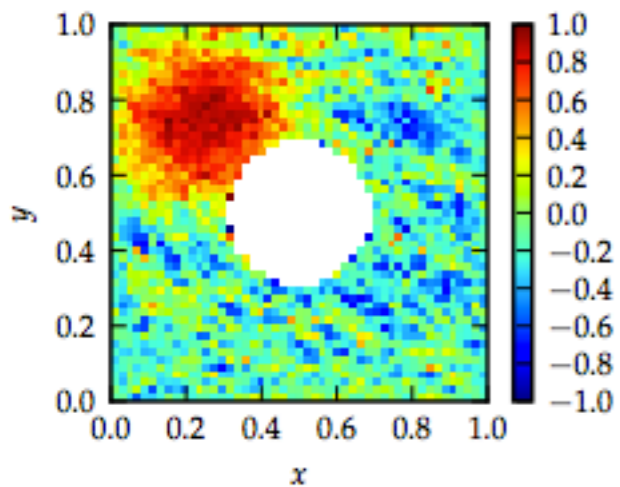


is there a code? If so, does it have a detectable shape?

yup



START



recording
device

is there a code? If so,
does it have a
detectable shape?

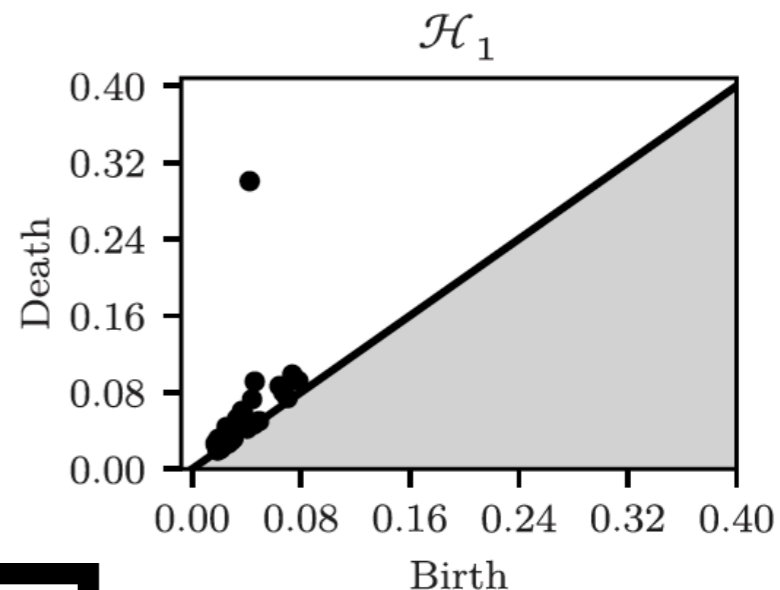
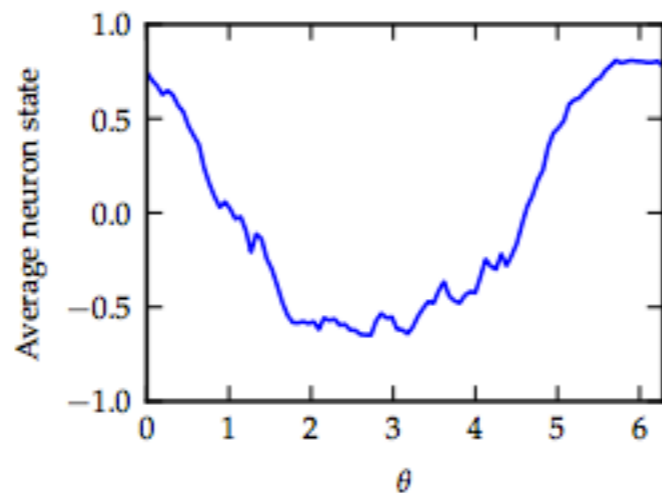
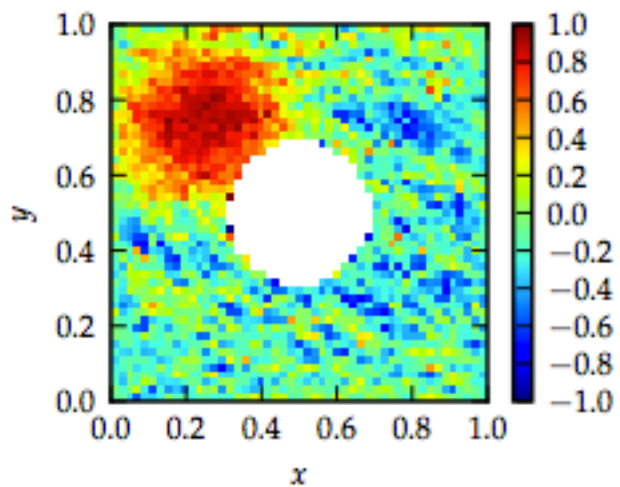
yup

SPACE

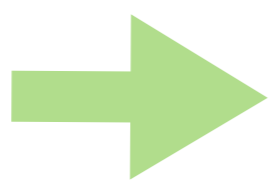
Find most likely feature

START

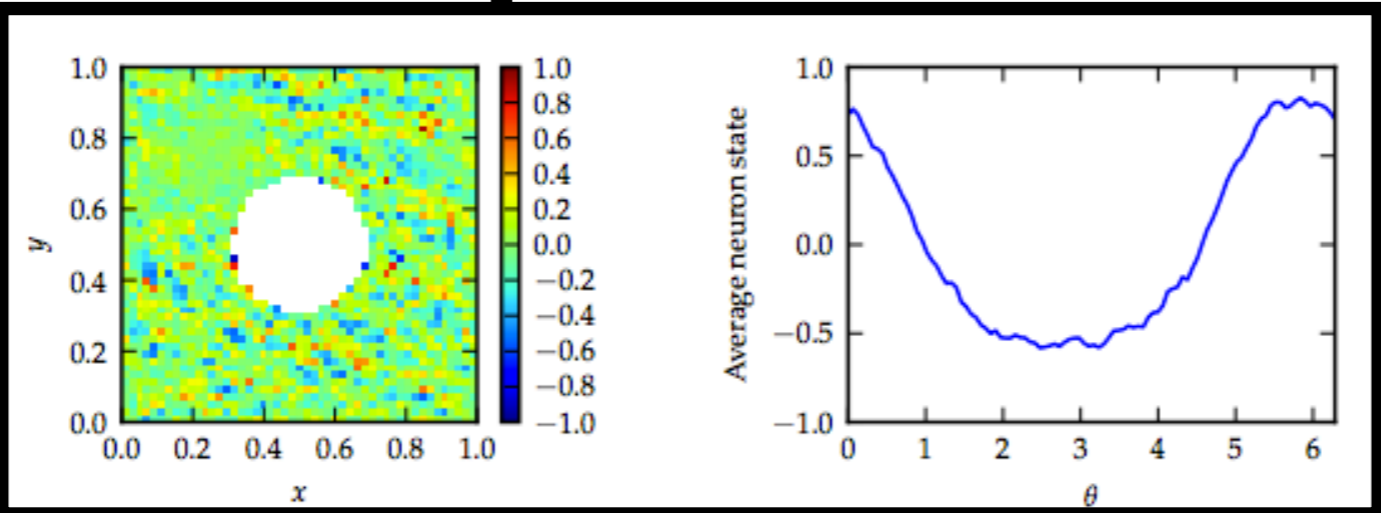




recording device



is there a code? If so, does it have a



yup



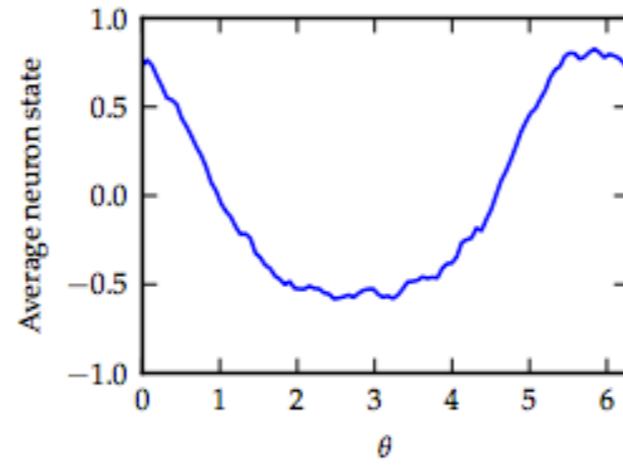
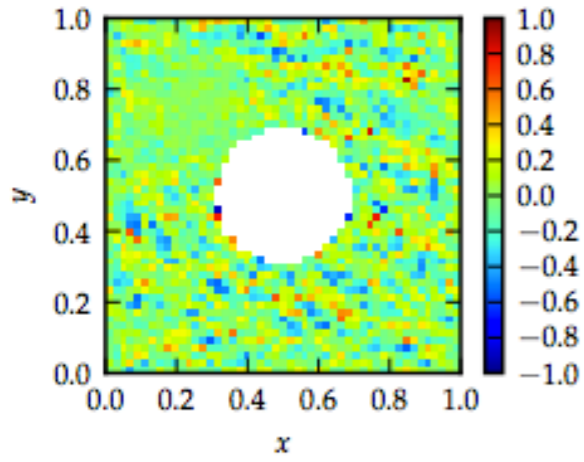
SPACE

Find most likely feature

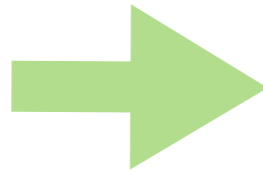
found something!
make "new" data
without feature

START

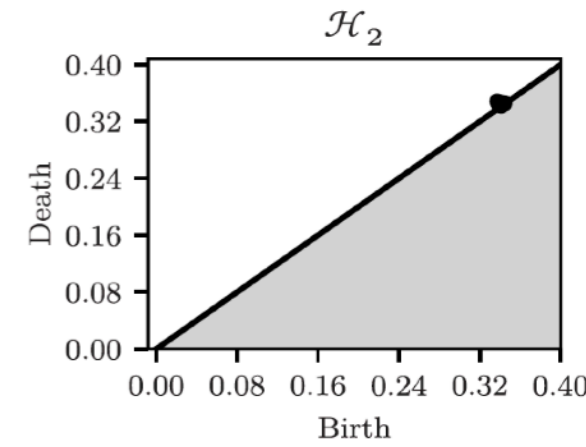
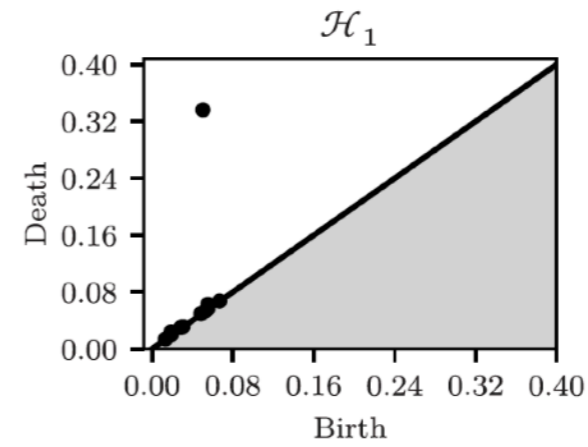
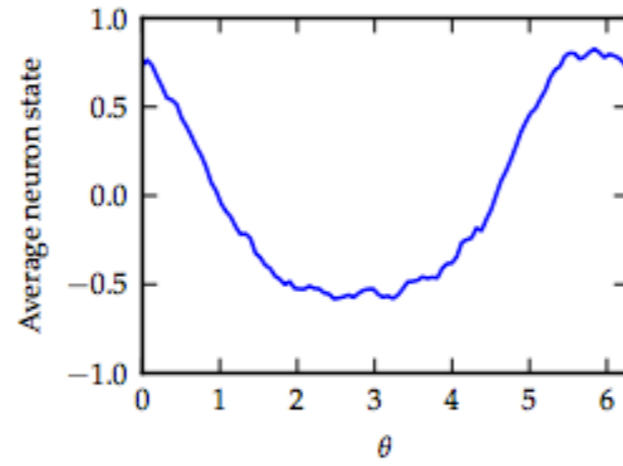
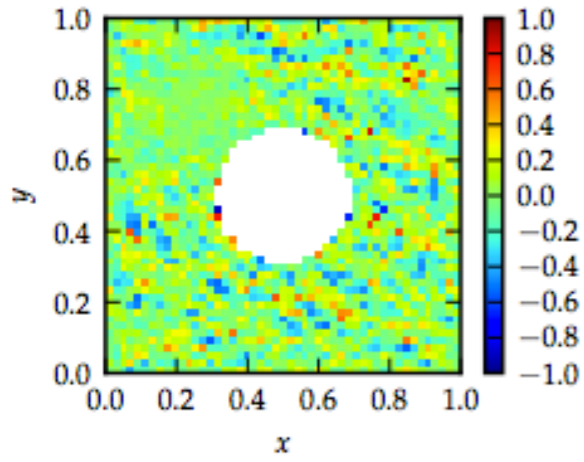




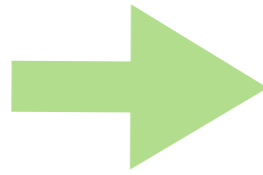
Residual
data



is there a code? If so,
does it have a
detectable shape?

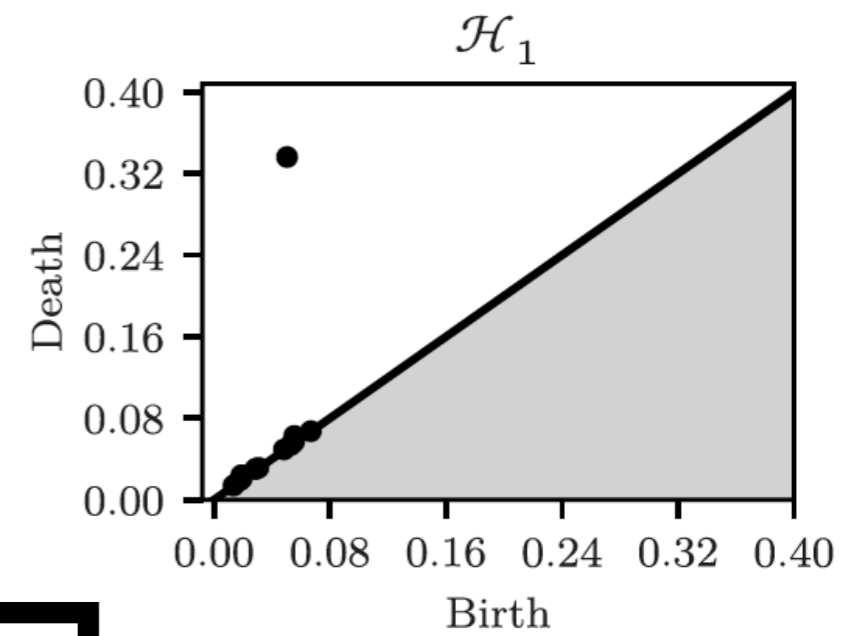
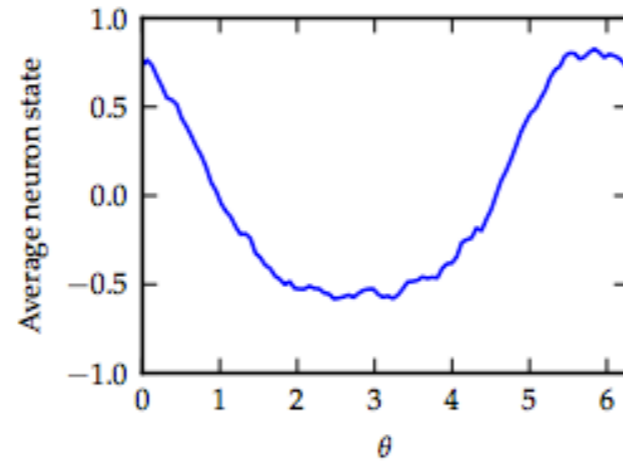
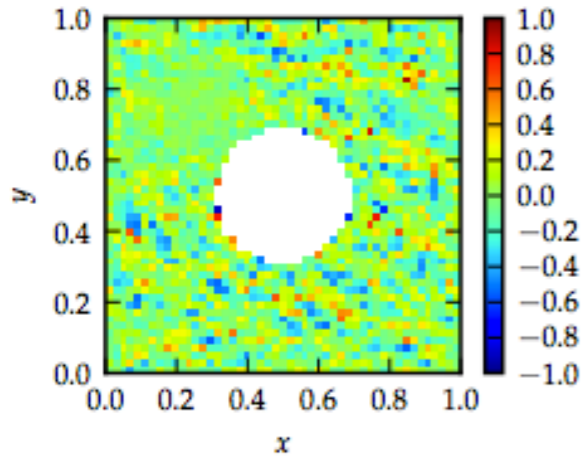


Residual
data

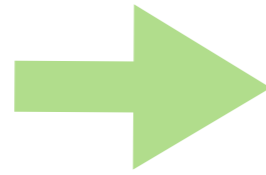


is there a code? If so,
does it have a
detectable shape?

yup



Residual
data



is there a code? If so,
does it have a
detectable shape?

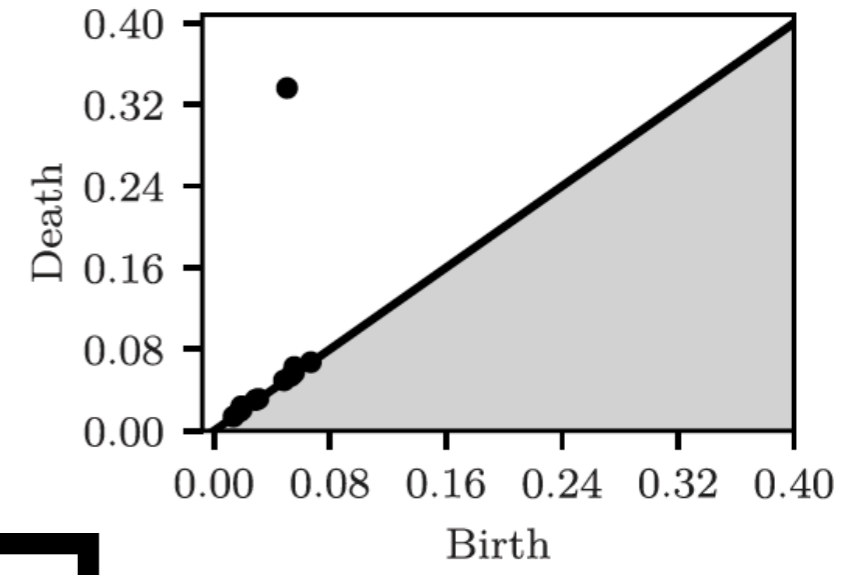
yup



Head direction

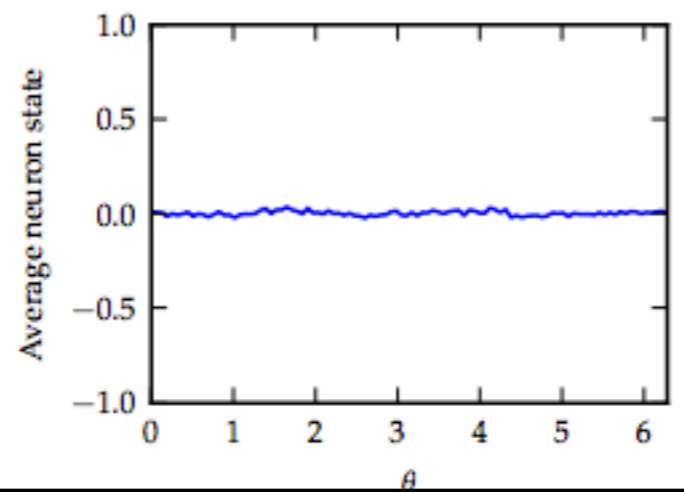
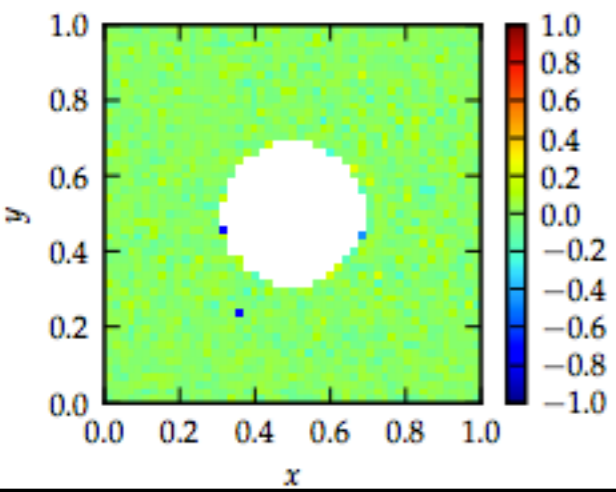
Find most likely feature

\mathcal{H}_1



Residual

is there a code? If so,



type?

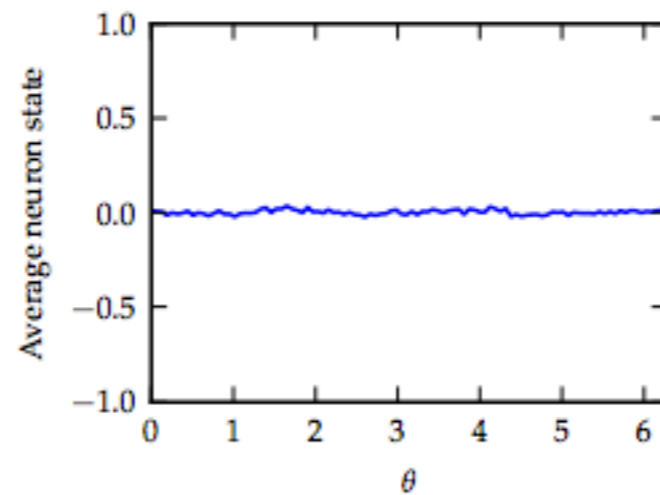
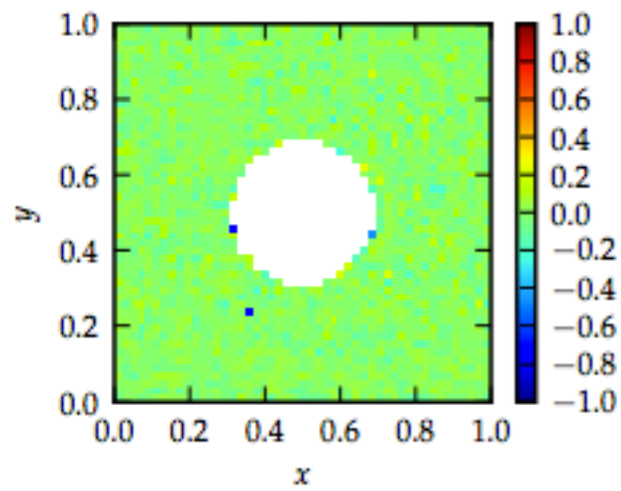
yup



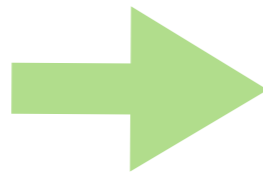
Head direction

found something!
make "new" data
without feature

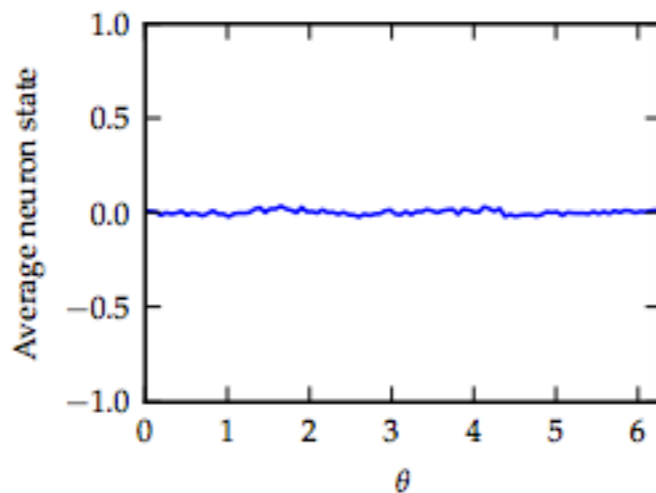
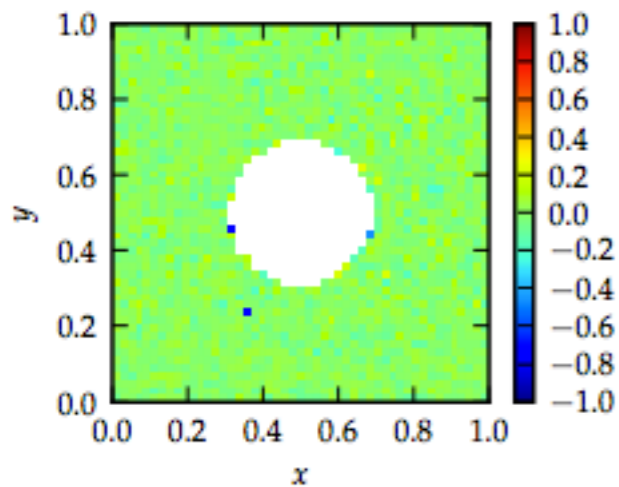
Find most likely feature



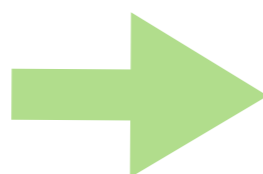
Residual
data



is there a code? If so,
does it have a
detectable topology?

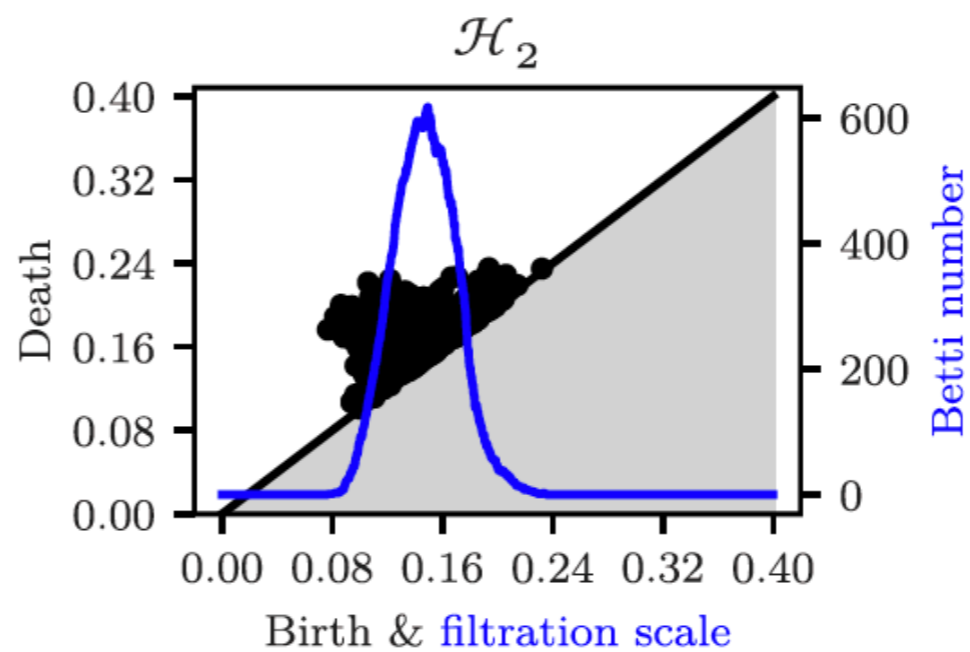
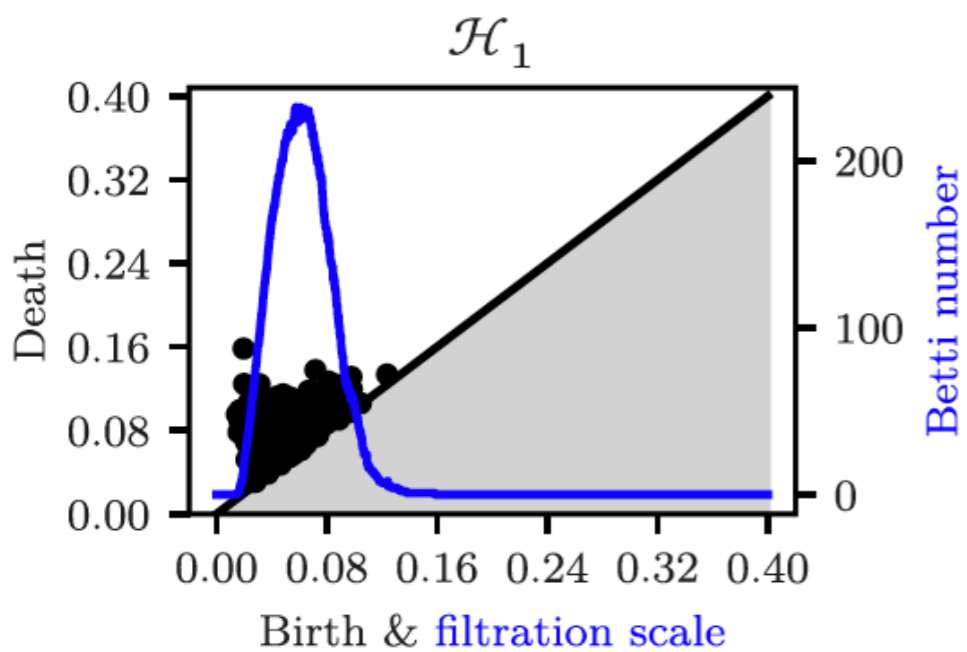
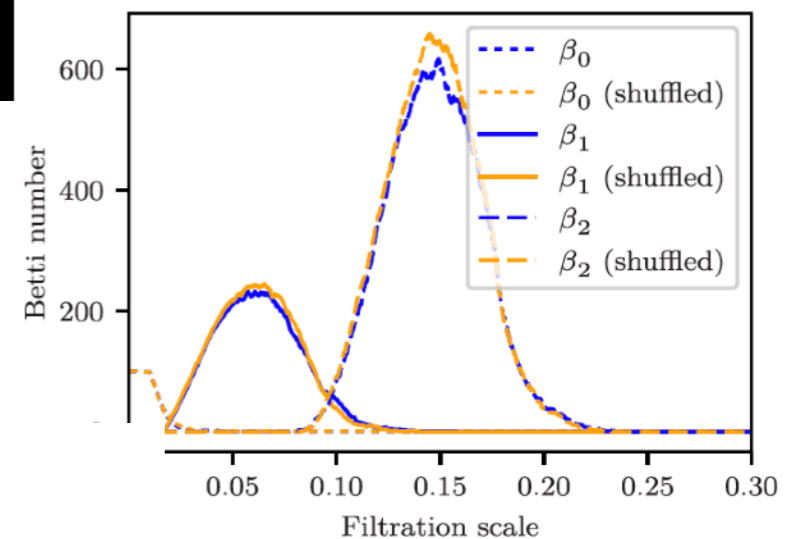


Residual data



is there a code? If so, does it have a detectable topology?

nope (all done)



So, this approach let's us ask

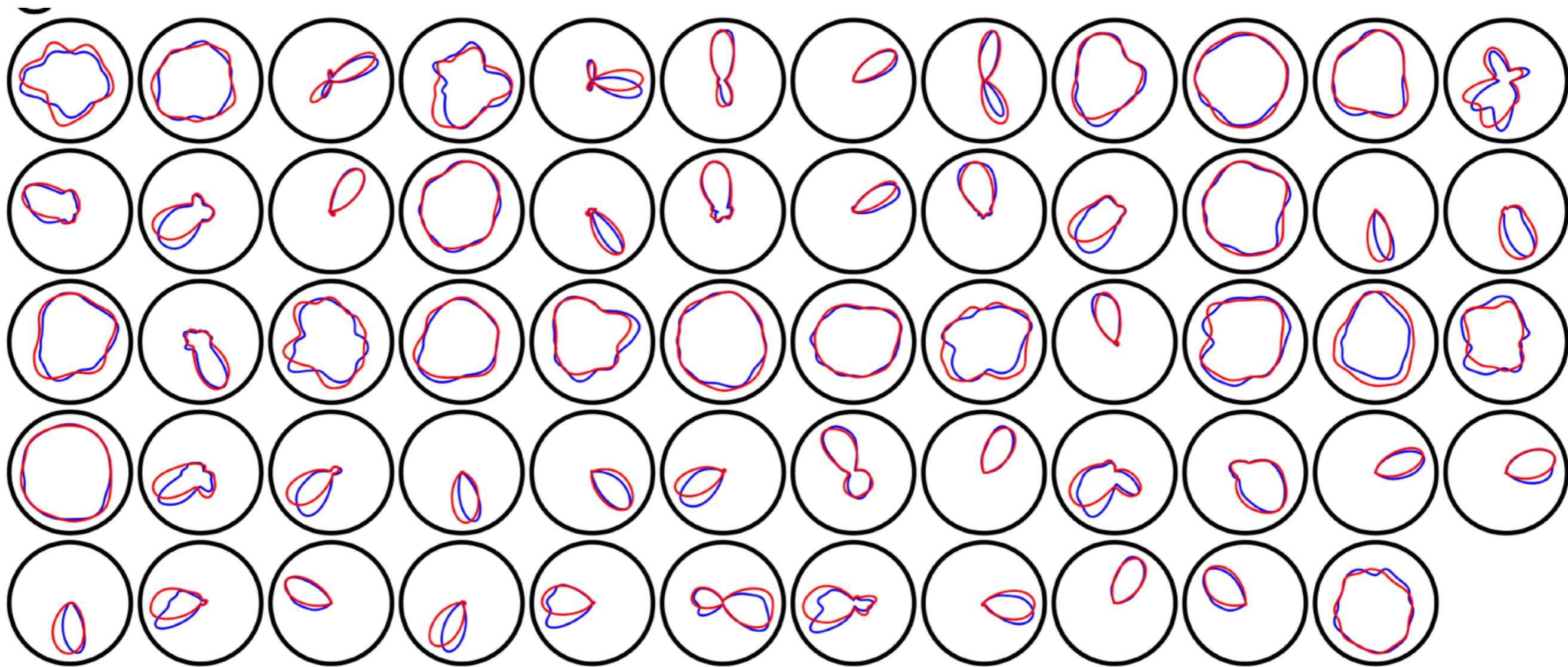
1) is there anything interesting here?

2) does it have a shape?

3) is there anything else?

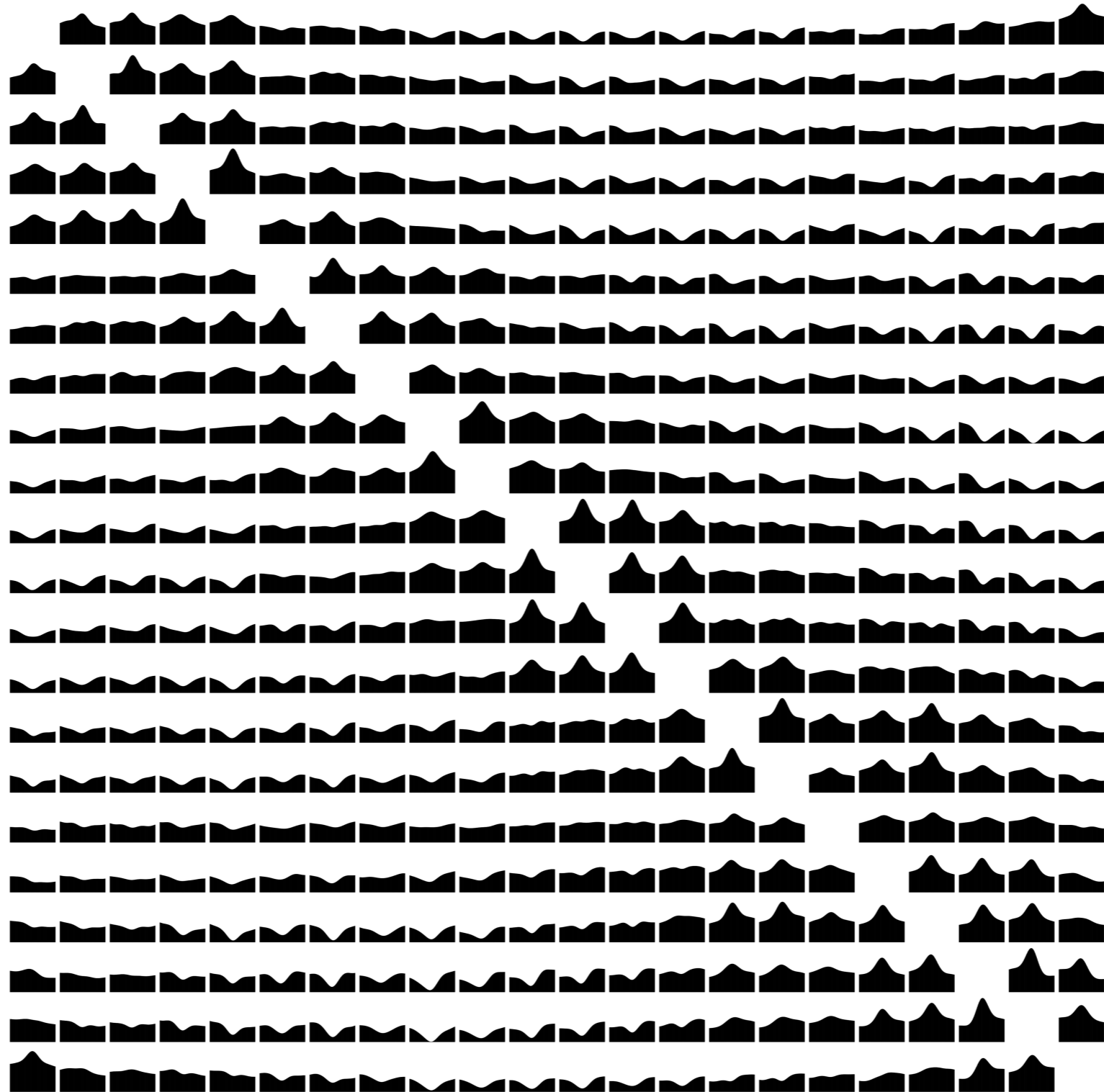
Great, so now for real data

Head direction neurons

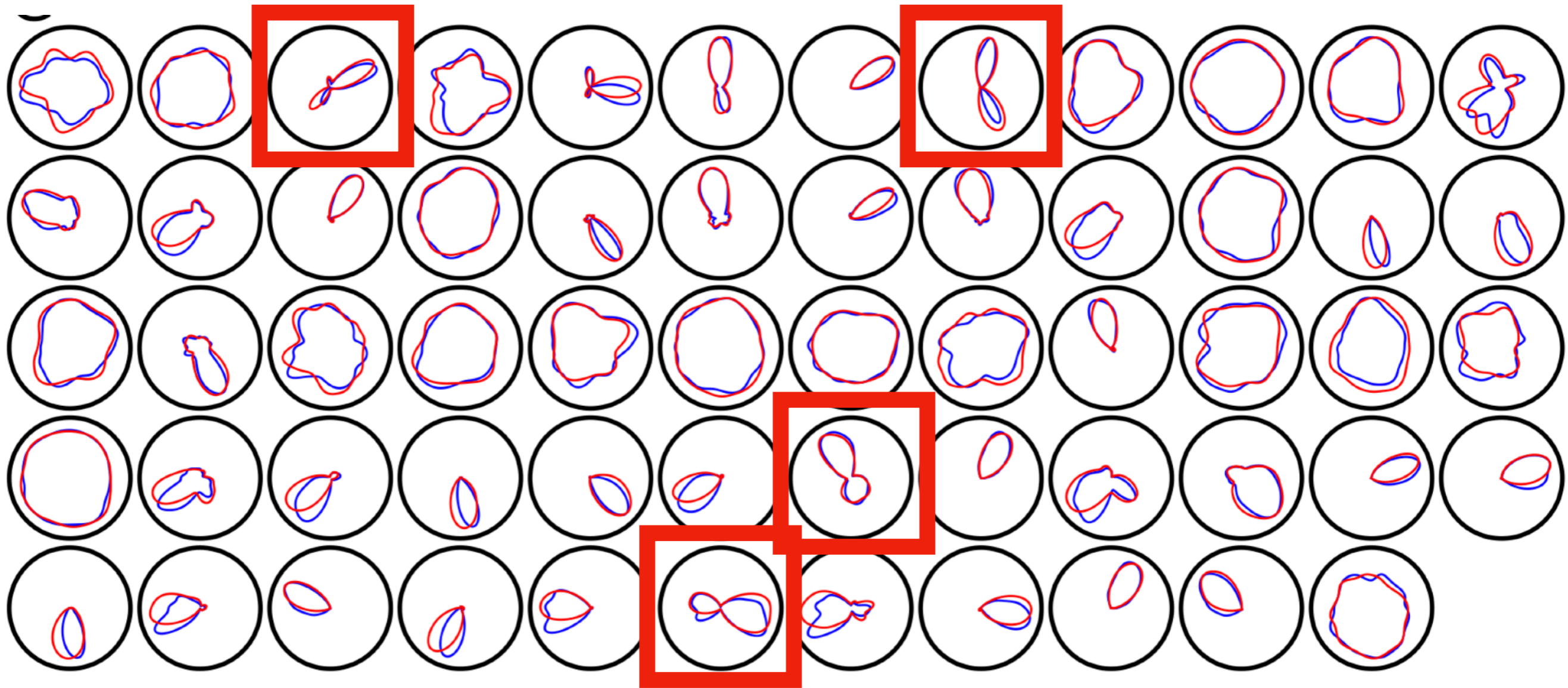


Data from Peyrache et al., 2015

If we choose the neurons properly the circle is very obvious using same as before

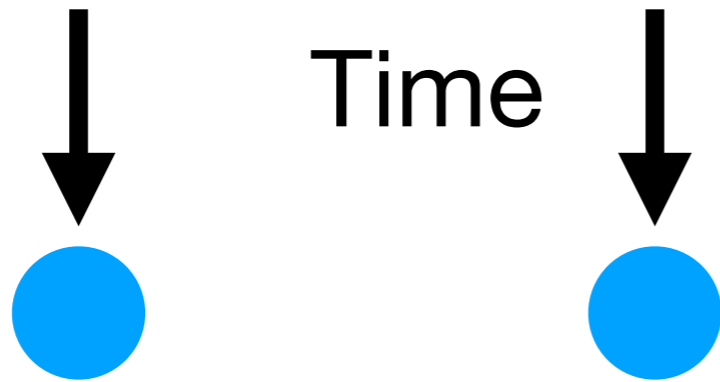
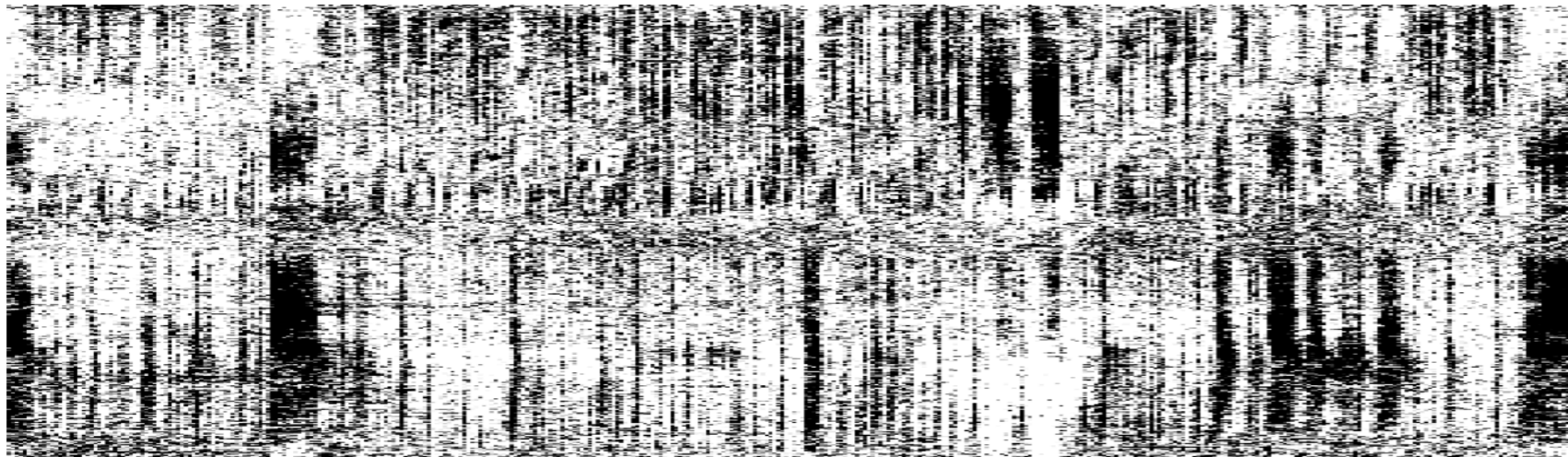


But some complicate things



Population states

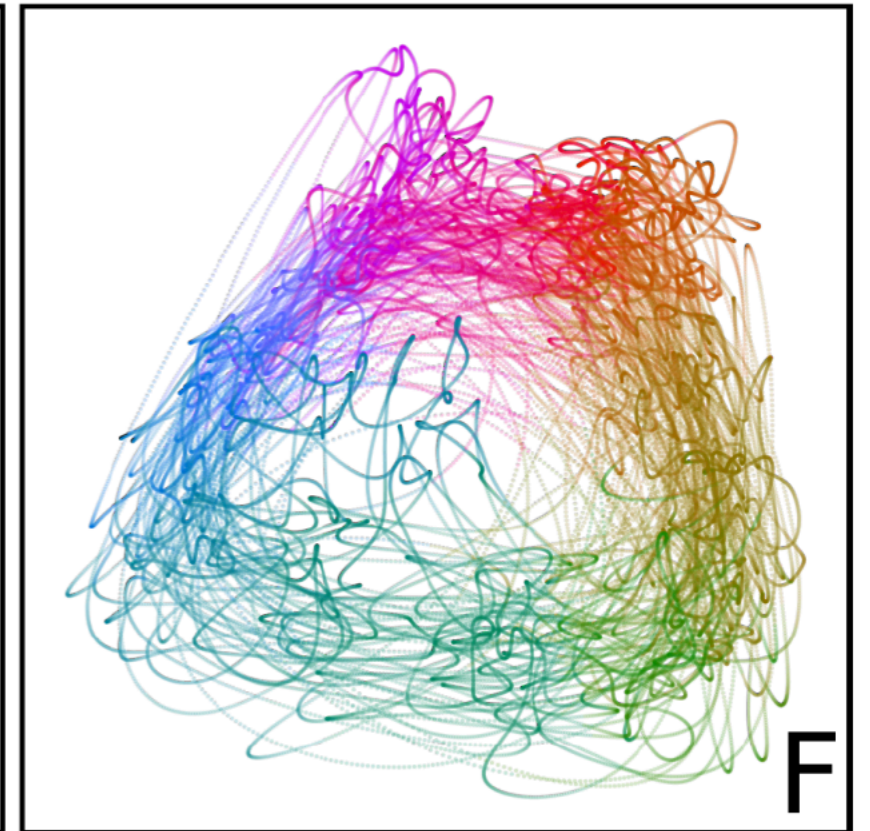
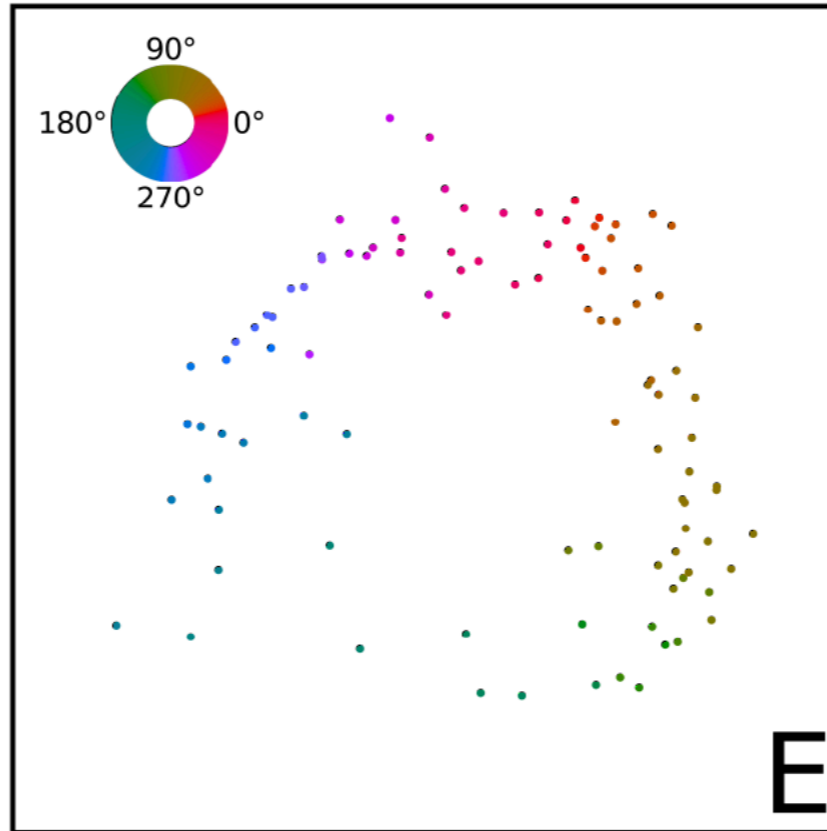
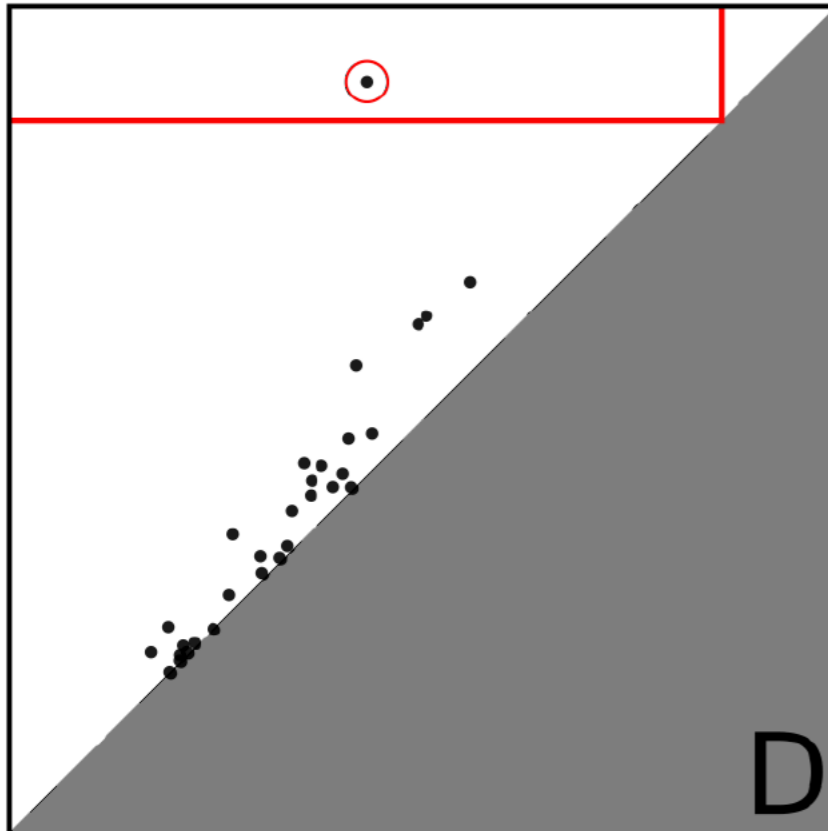
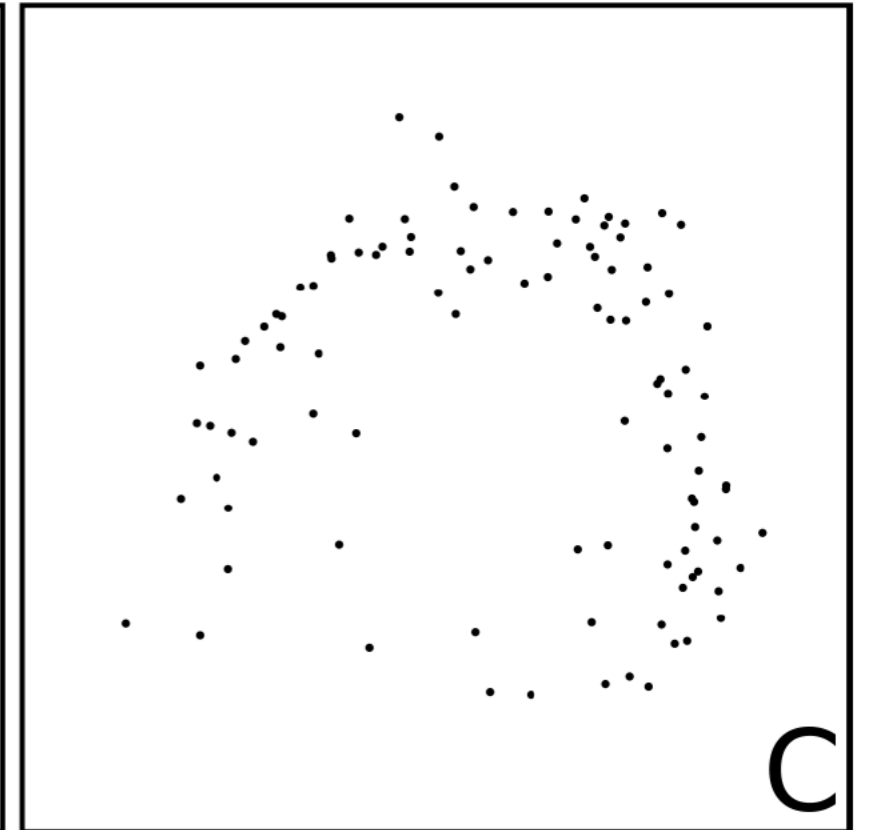
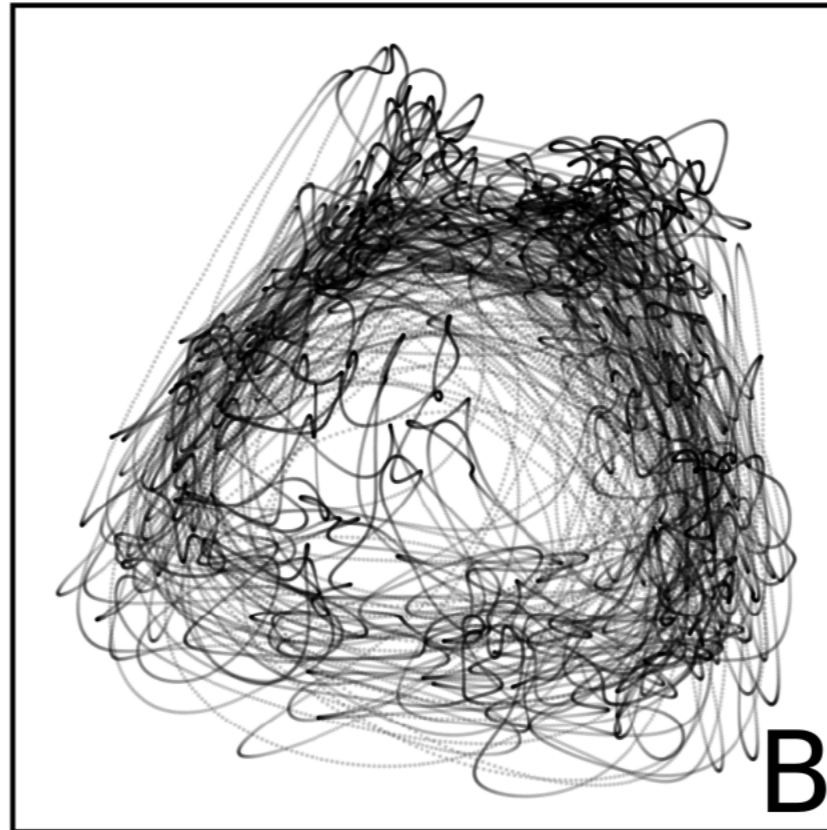
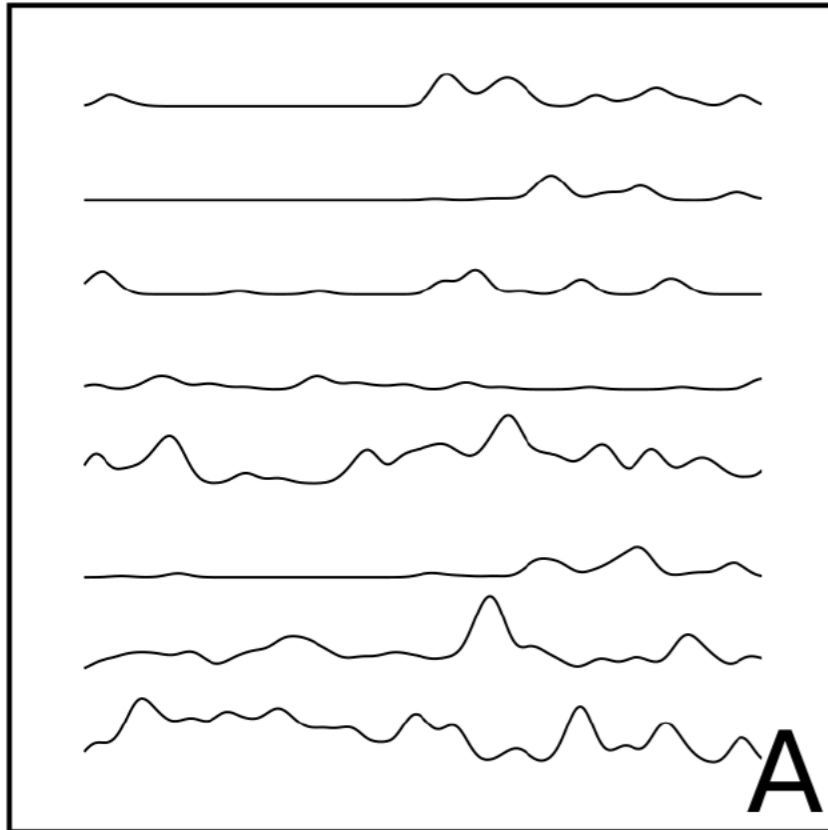
Neurons



**Firing rates
(250ms)**

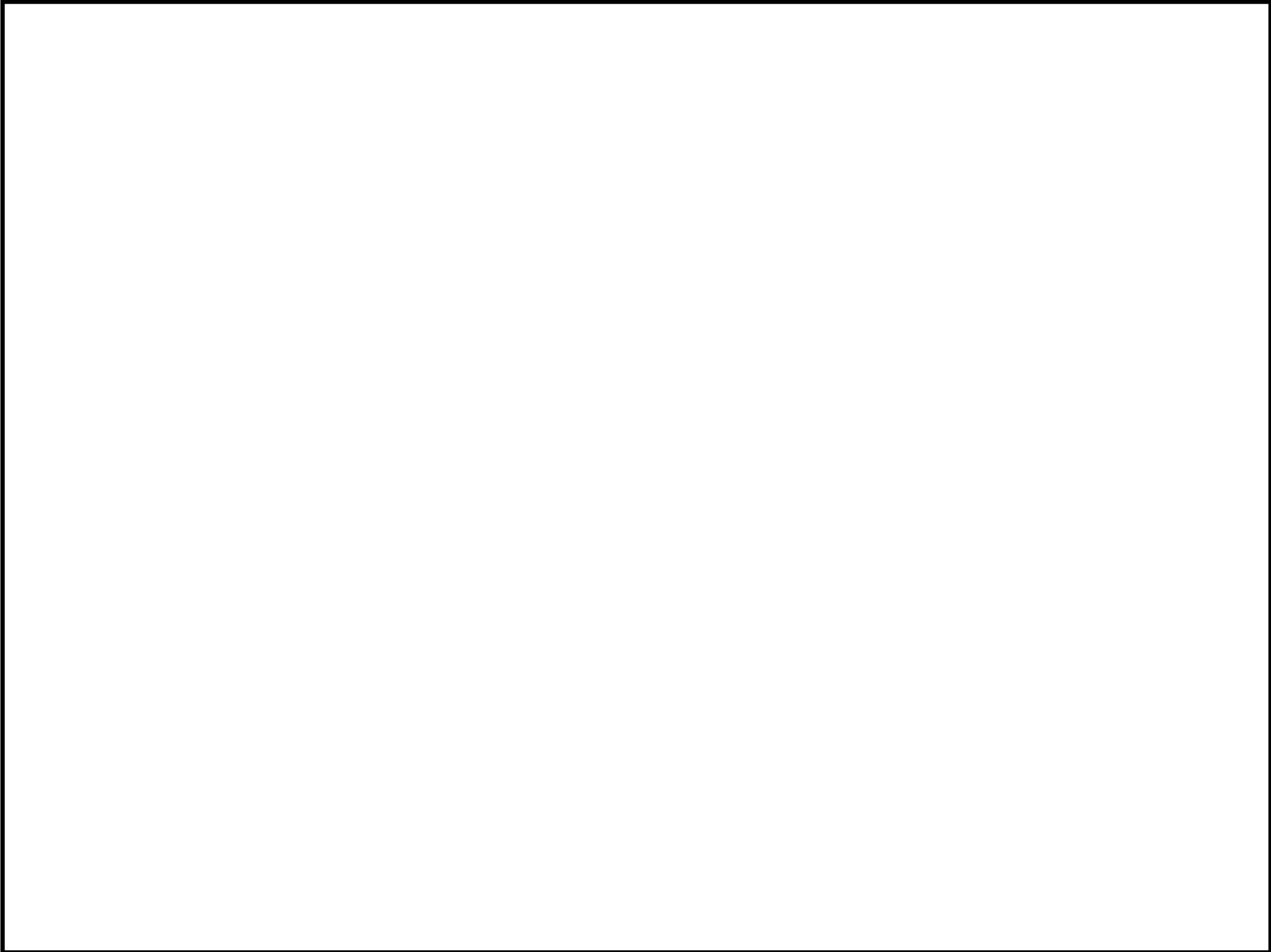
PCA

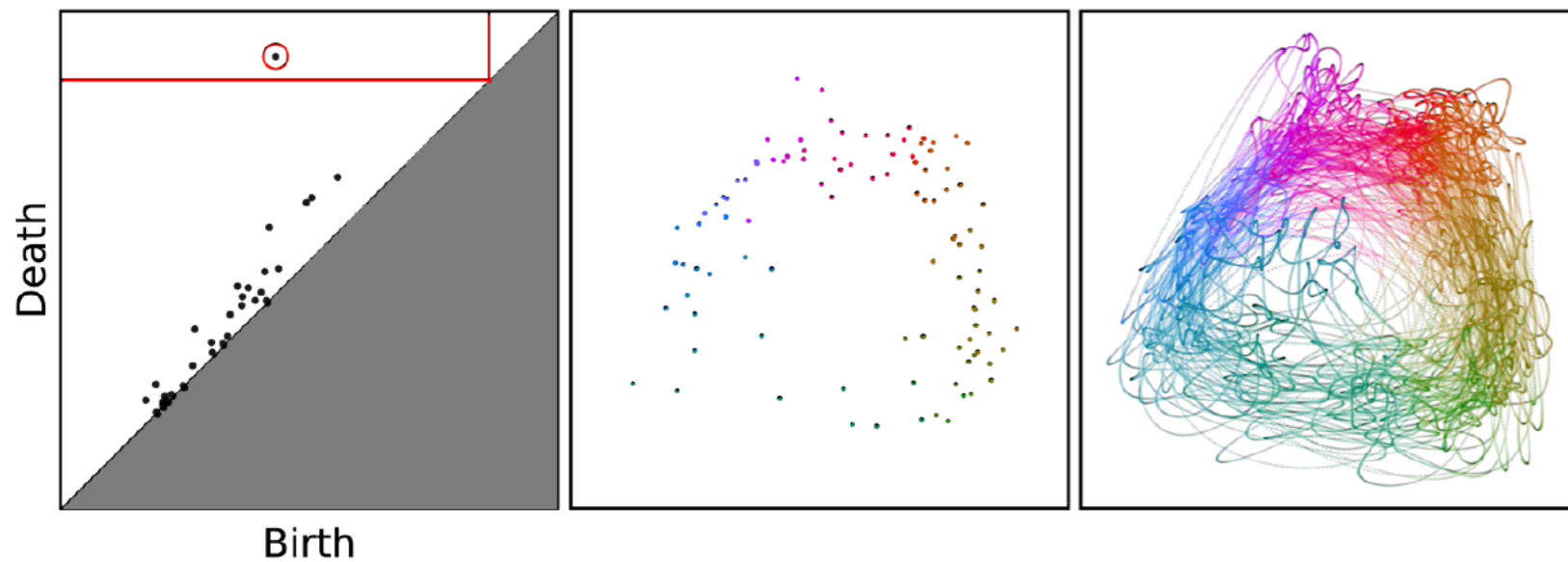
**Radial distance &
Topological denoising (Kloke & Carlsson, 2010)**



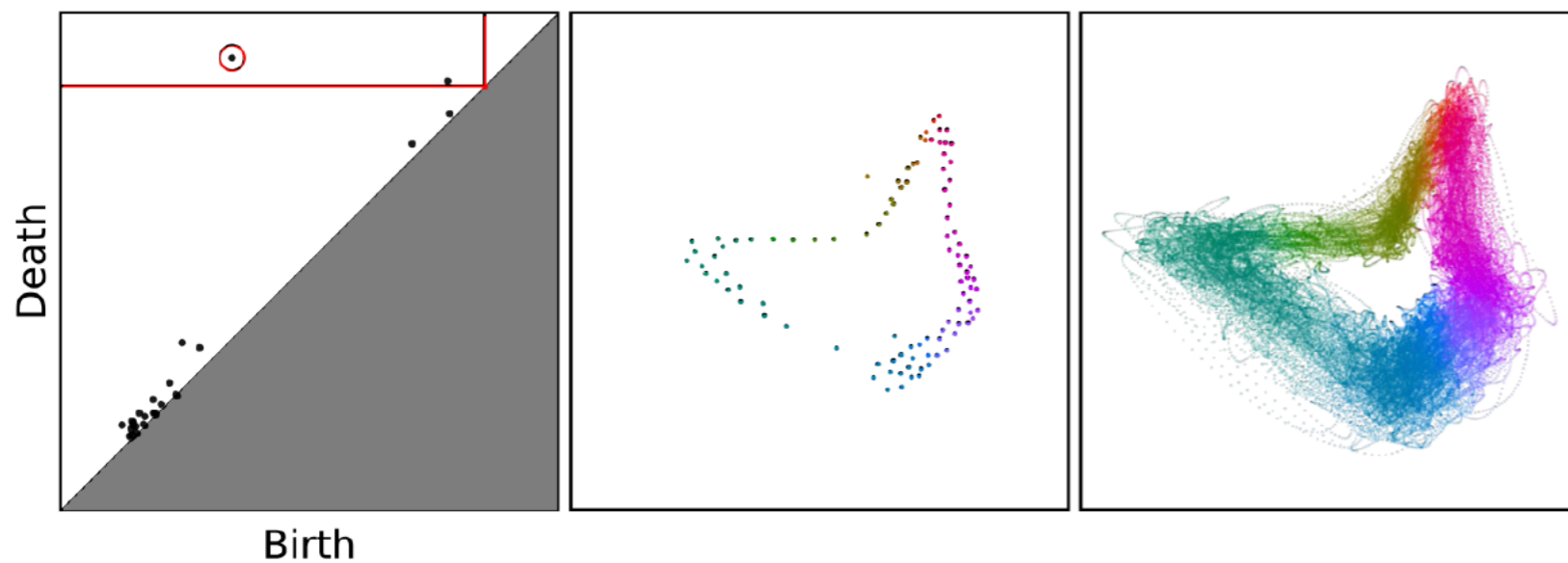
H1 Birth **Circular parametrization
de Silva et al., 2011**

**Match to
closest point**



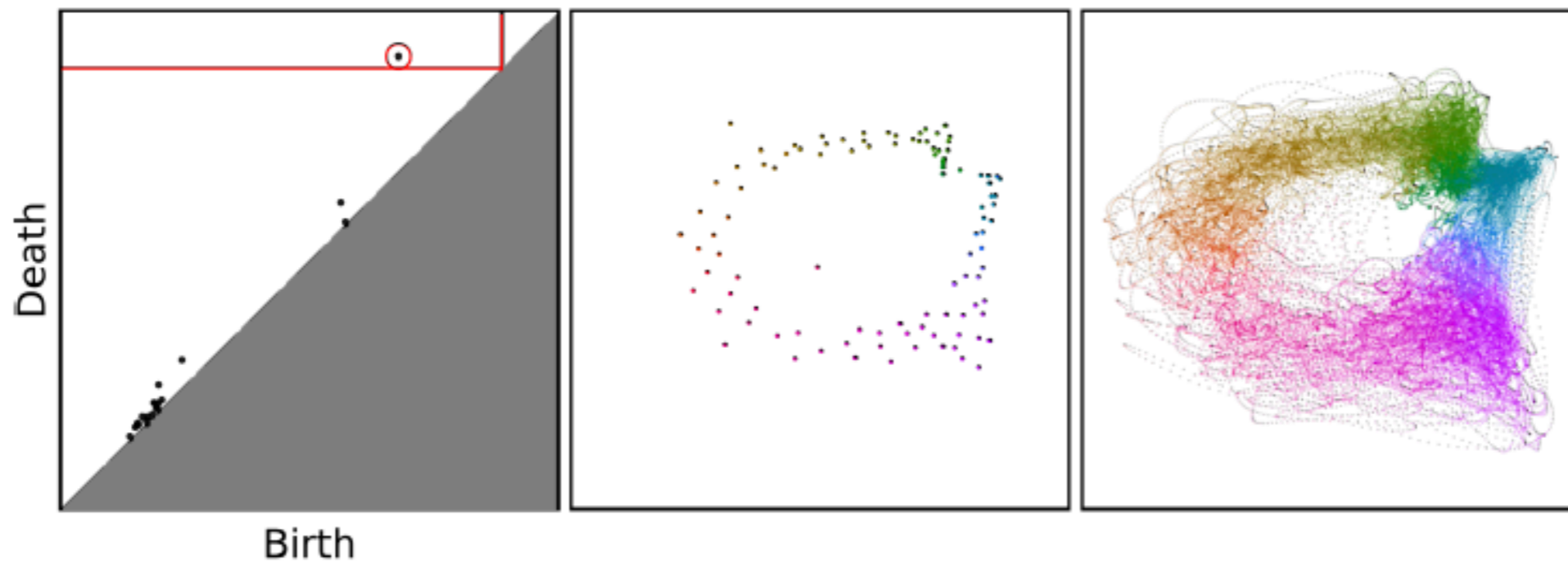
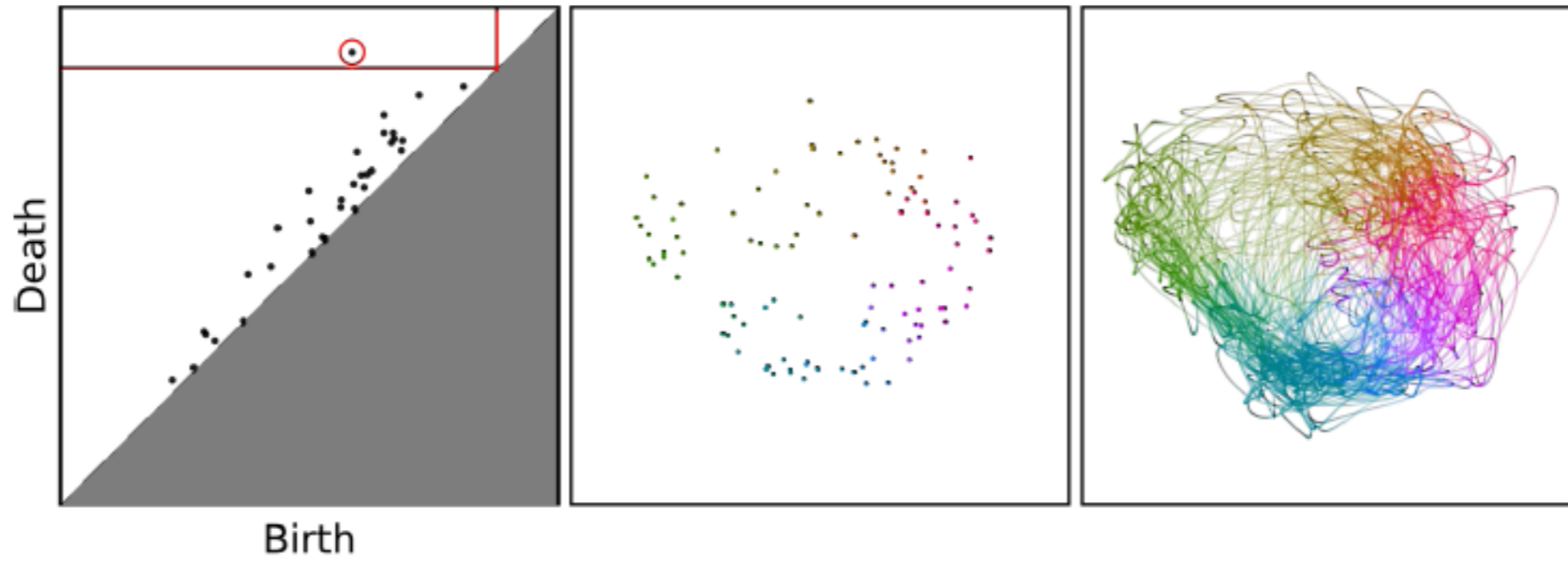


A circle from all neurons

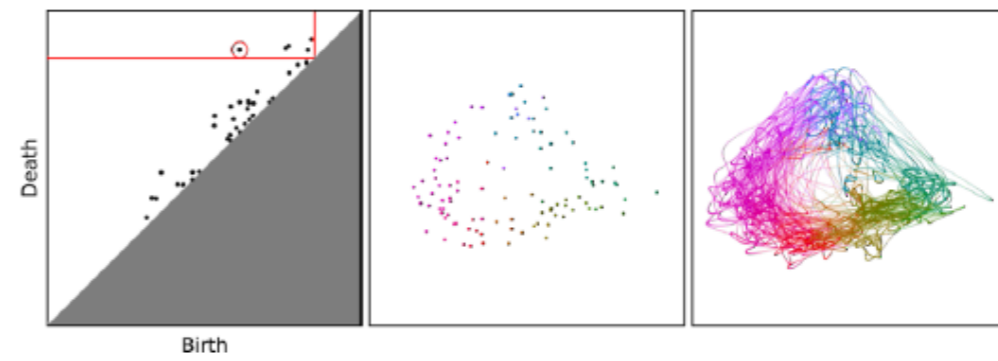
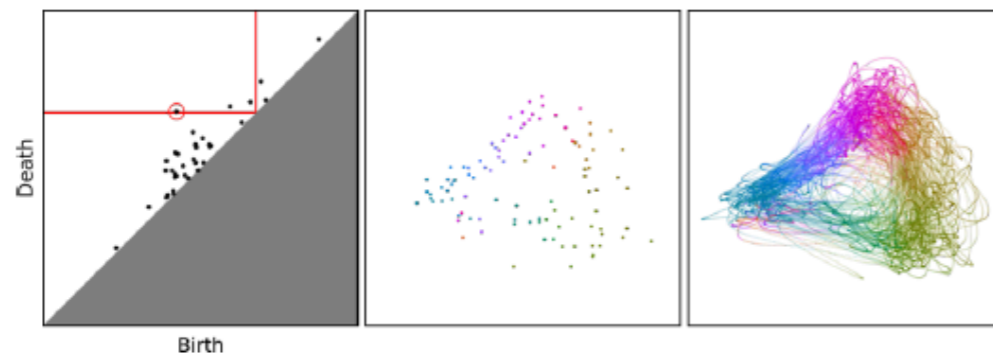
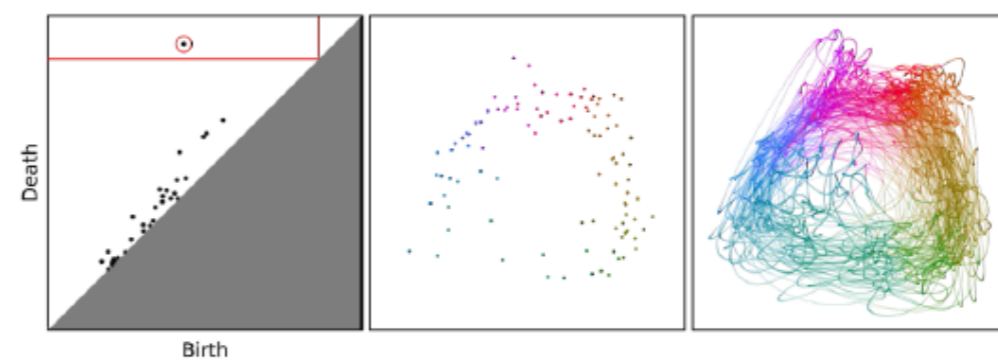
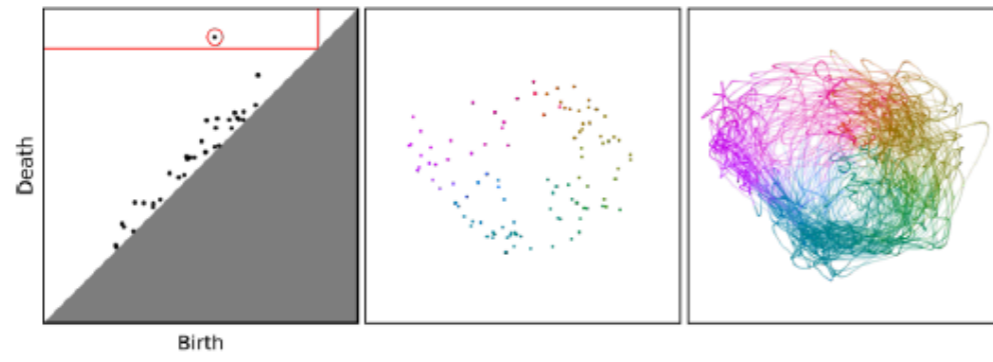
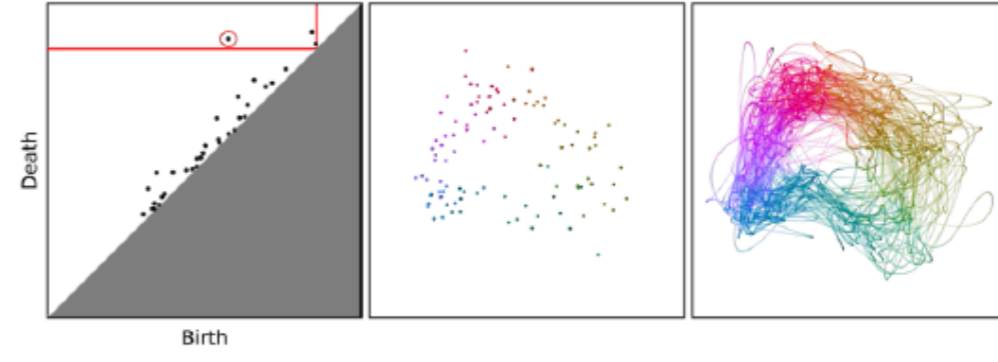
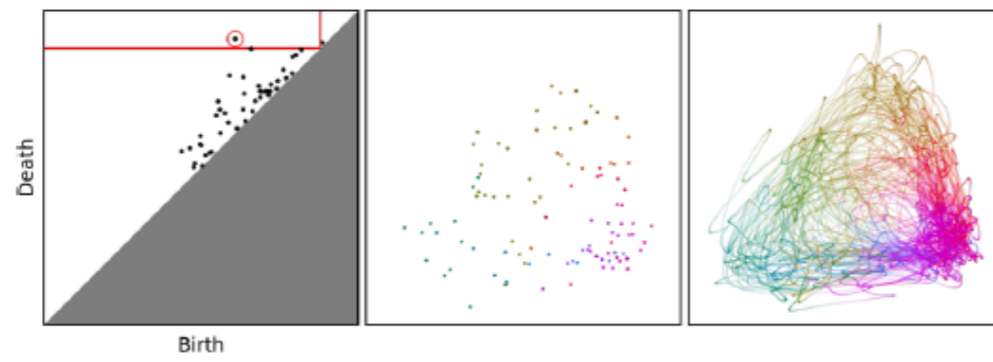
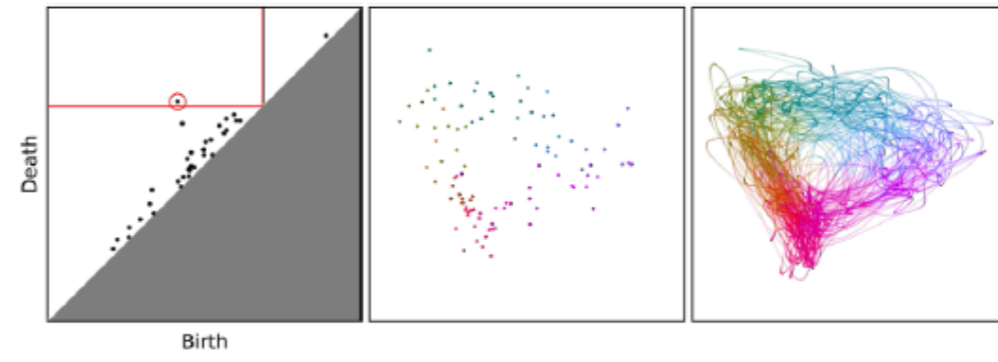
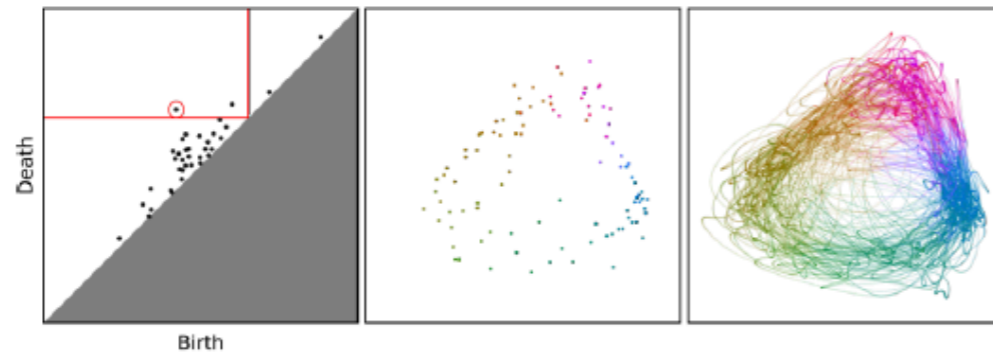


After removal of untuned neurons

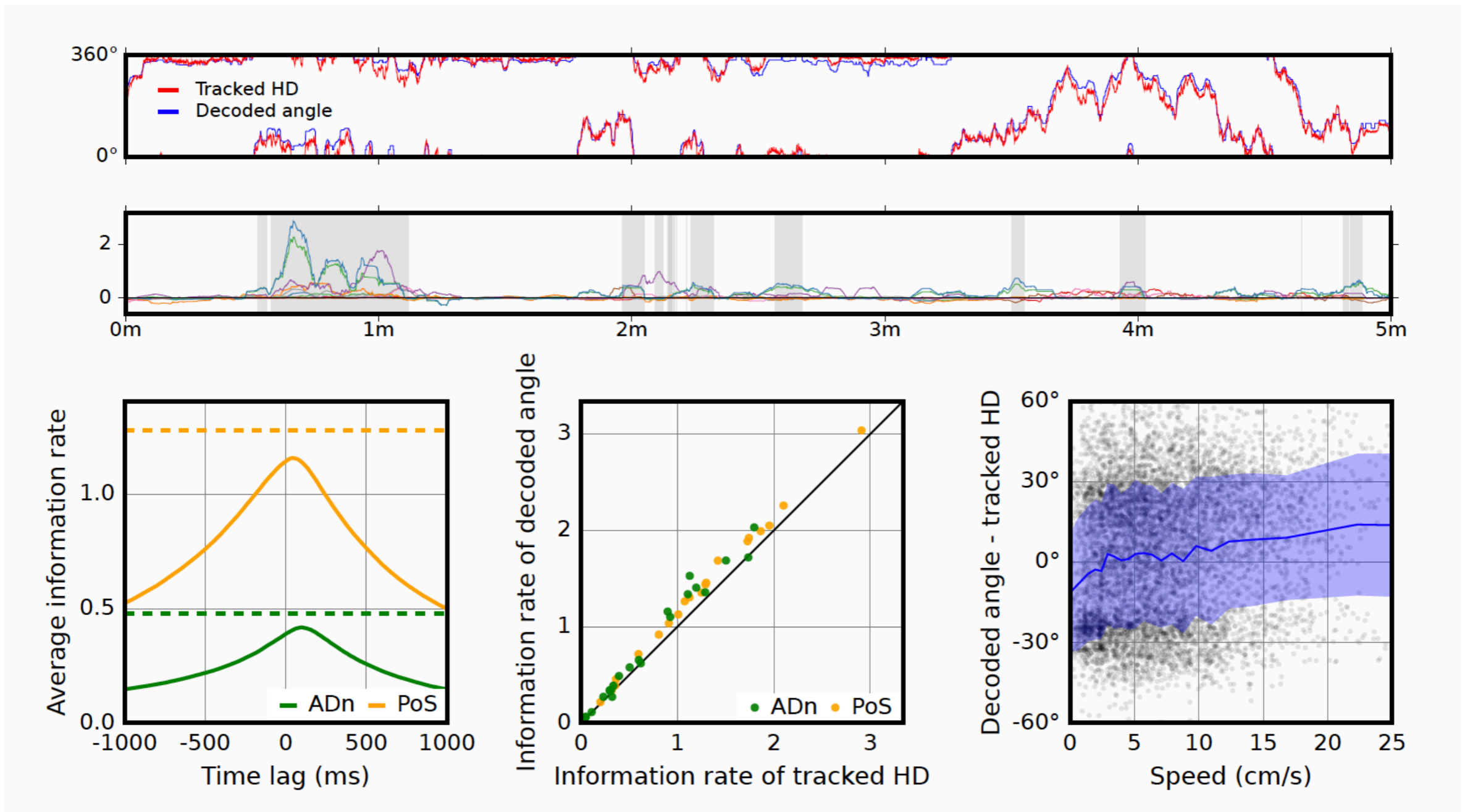
A tricky one



More examples



Comparison with tracked head direction



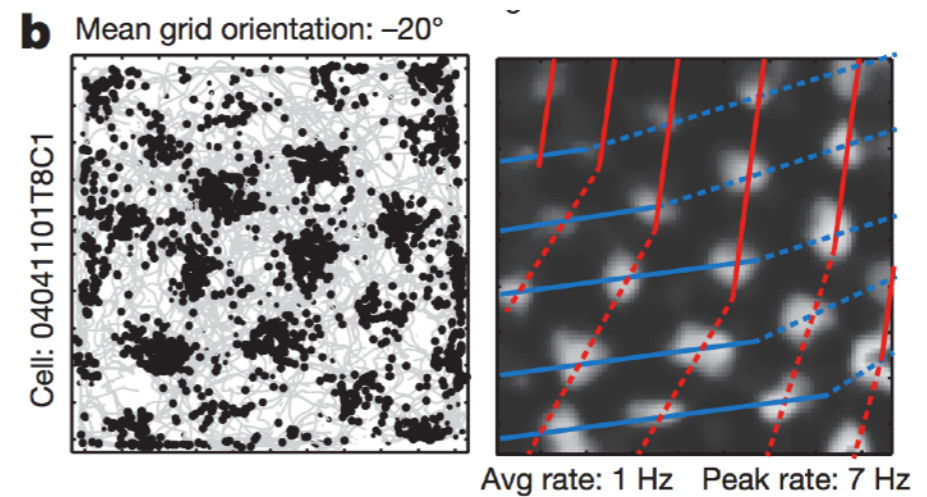
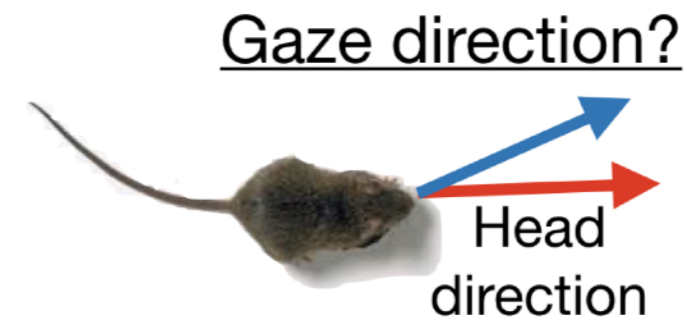
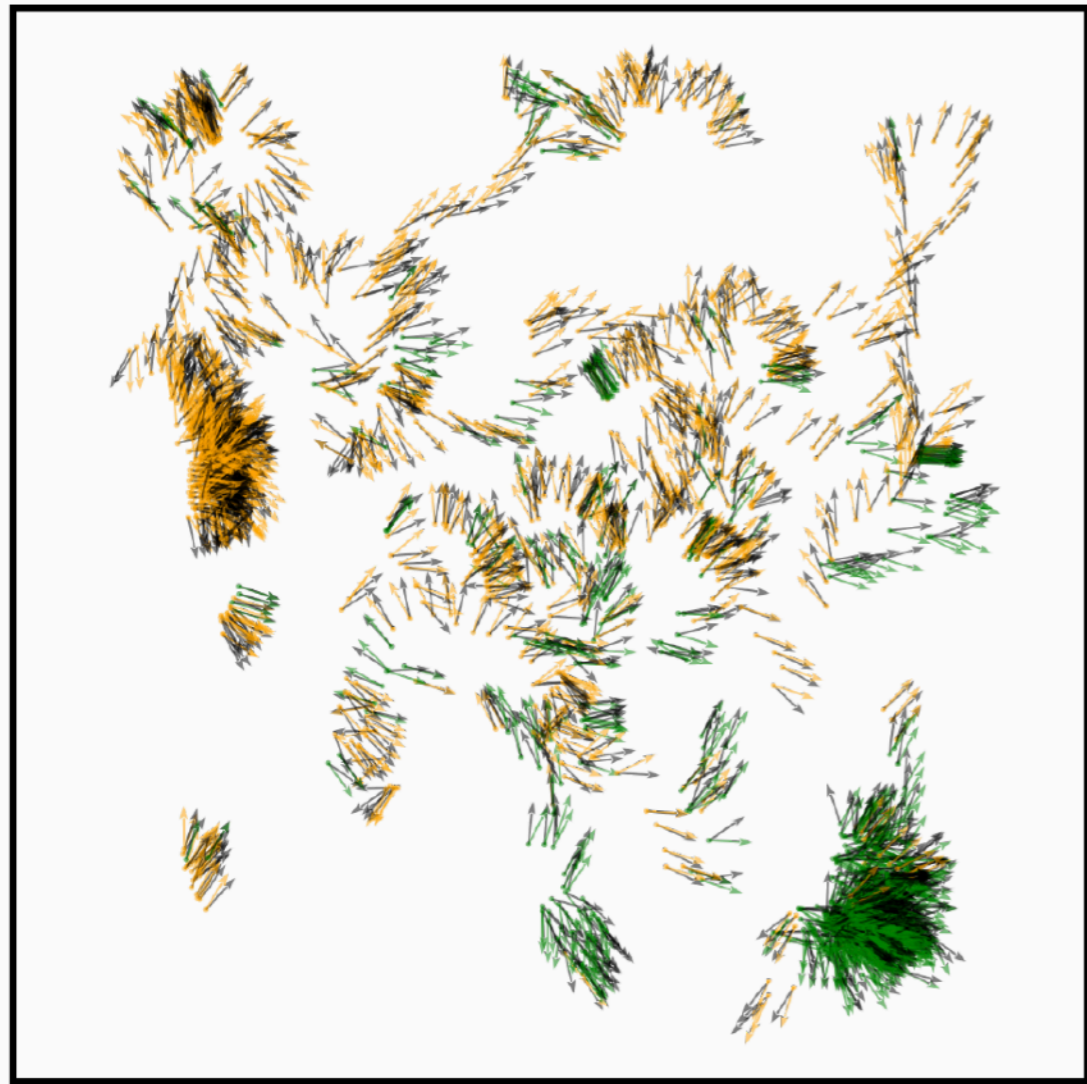
Comparison with tracked head direction



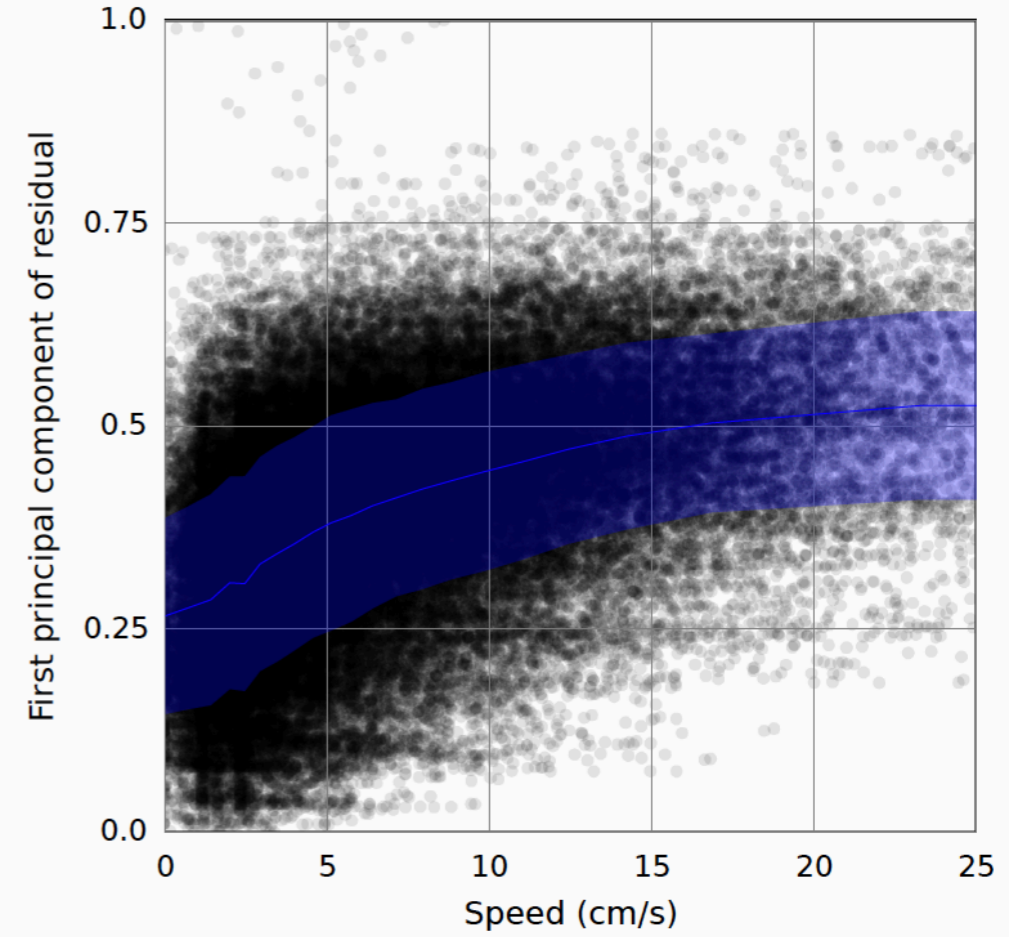
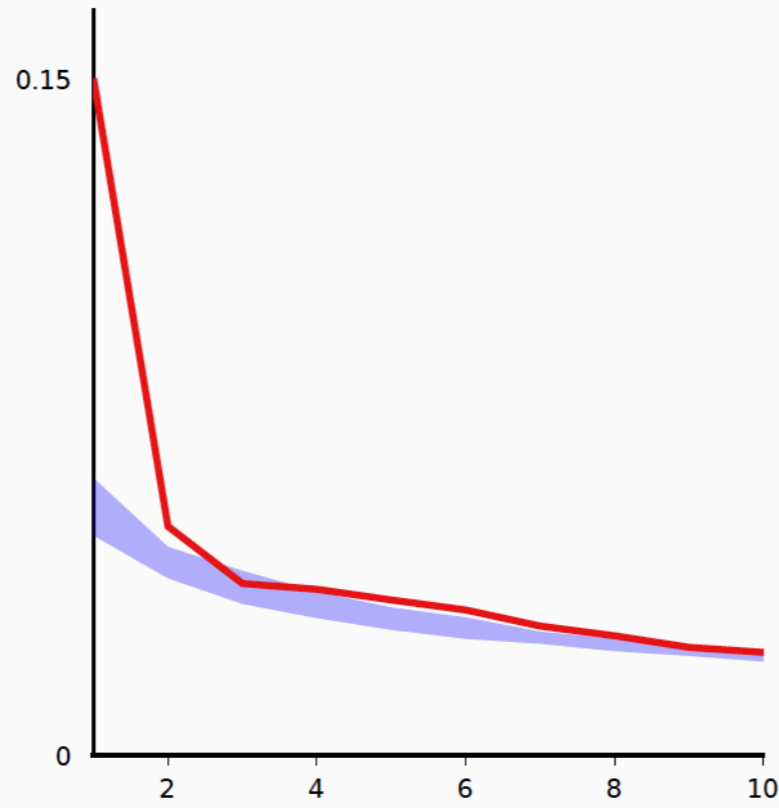
— Tracked tuning curves
— Decoded tuning curves

Comparison with tracked head direction

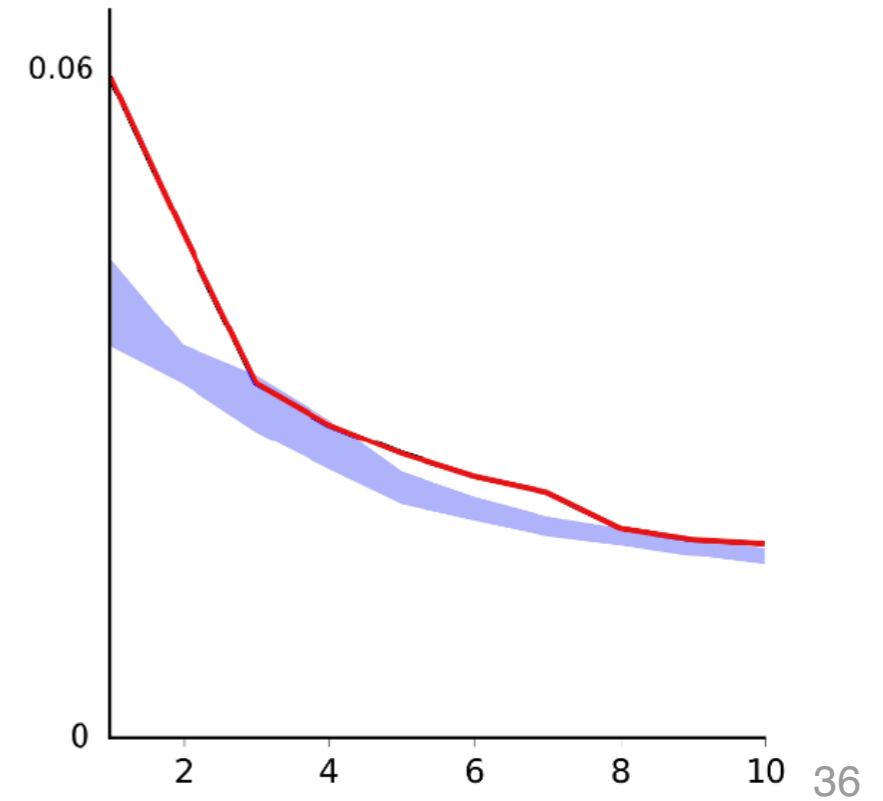
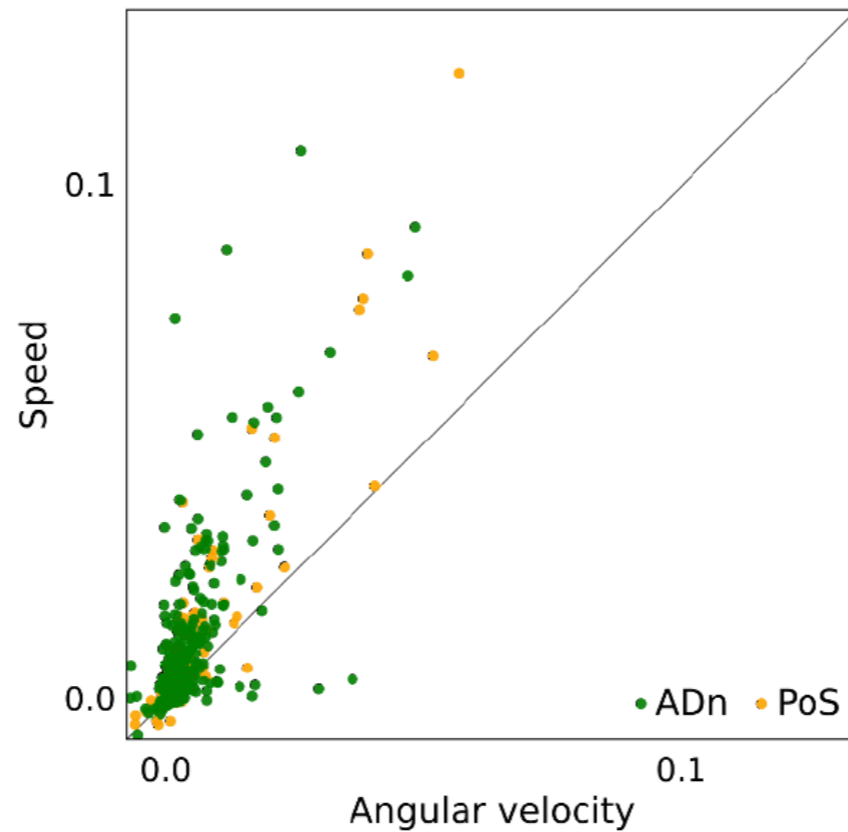
Unexpected benefit



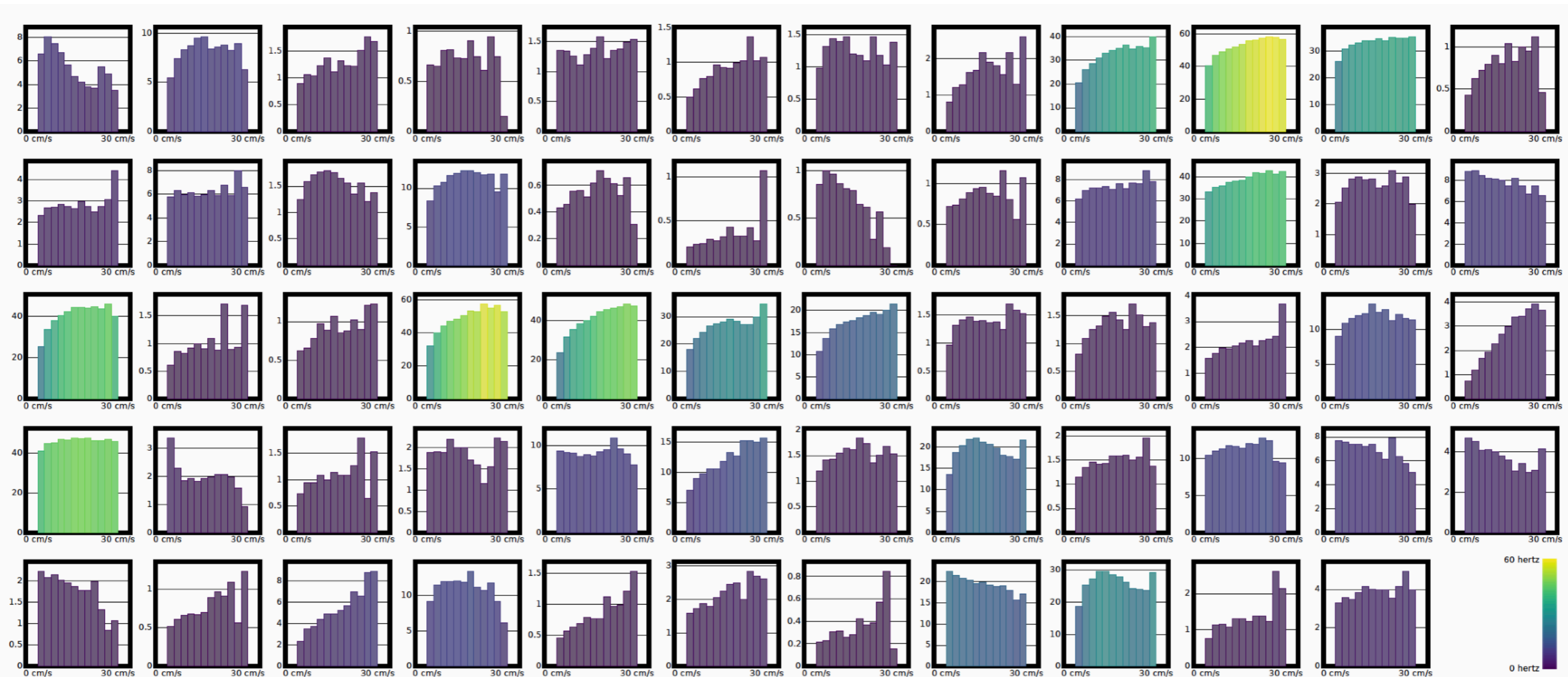
Looking at the residuals (as before)



Speed tuning stronger than angular velocity



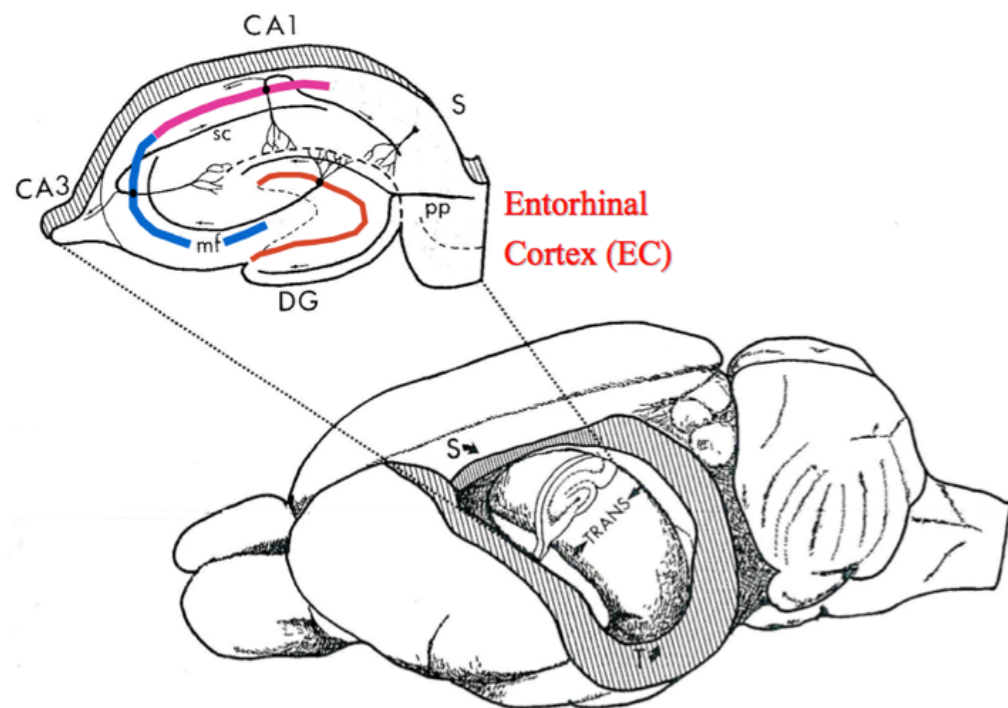
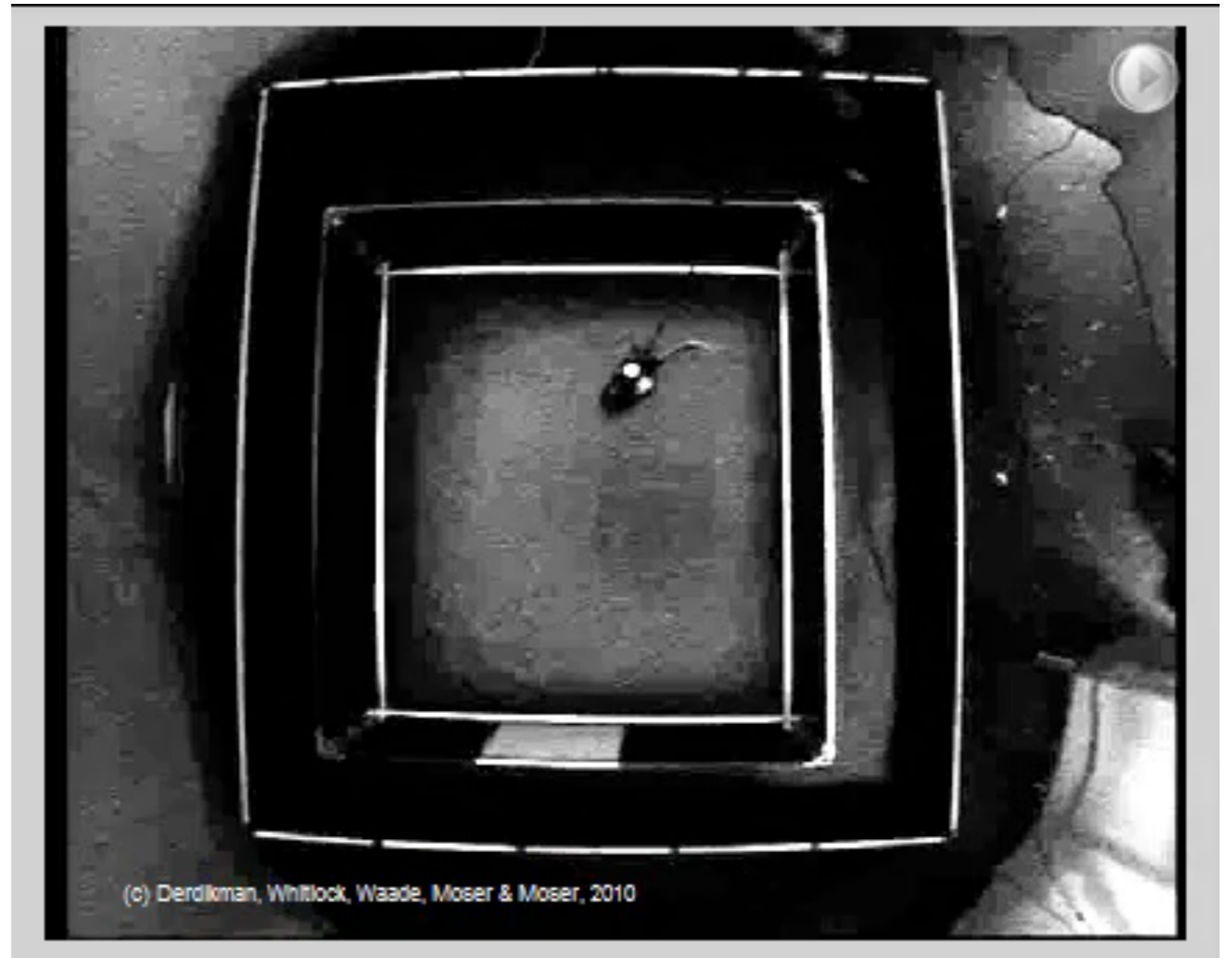
Example “speed” selectivity



Optimistic next steps

Grid cells

Fyhn et al., 2004,
Hafting et al., 2005

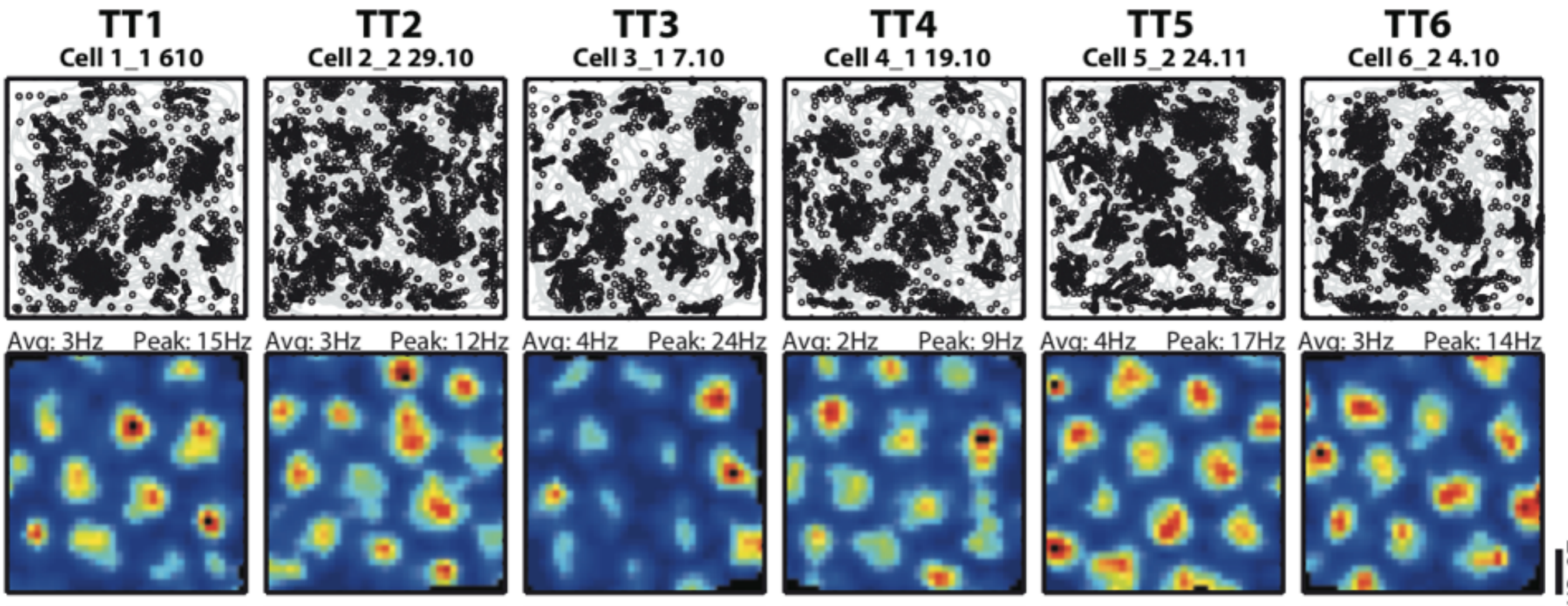


(Amaral and Witter 1989)

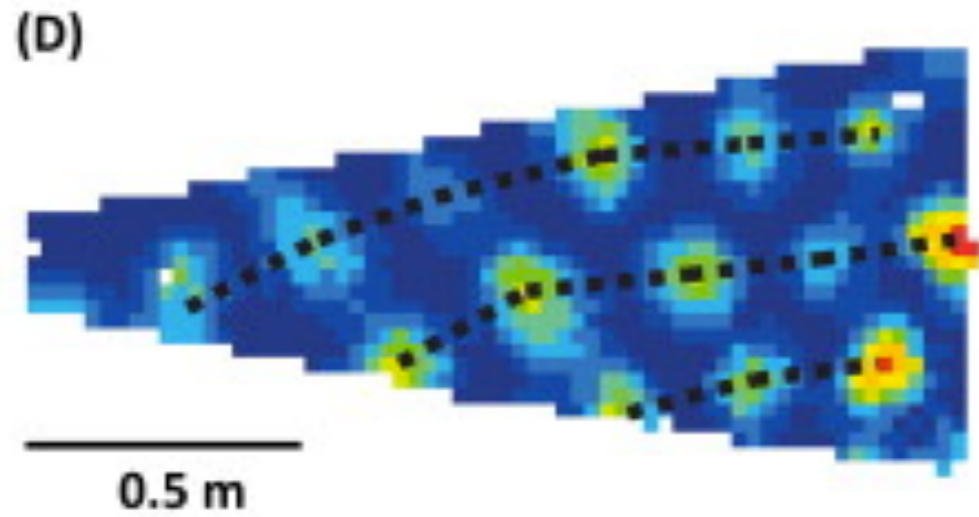
Would they have seen this
if it wasn't location???

Grid cell modules

Grid cells group according to their spacing and orientation, forming modules that behave coherently

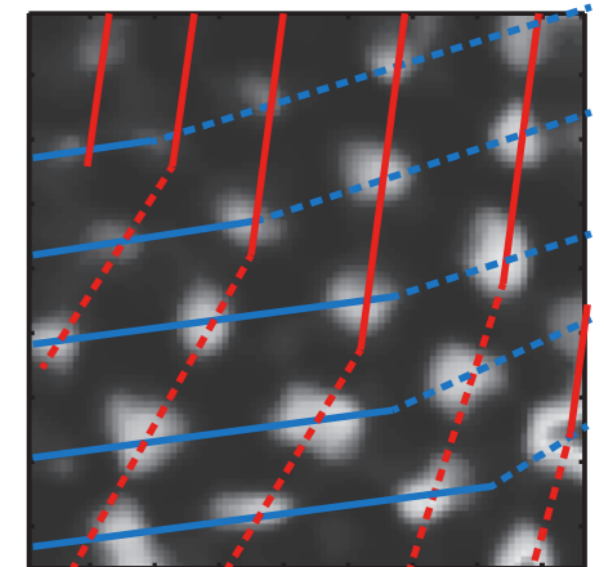
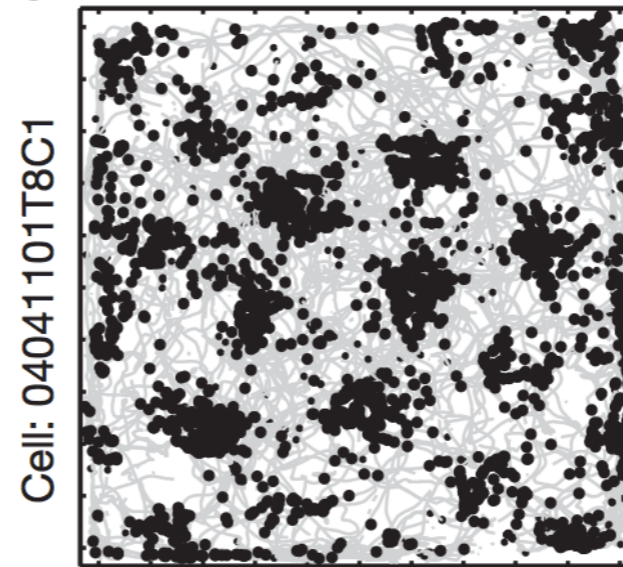


Other examples



Krupic et al, 2015

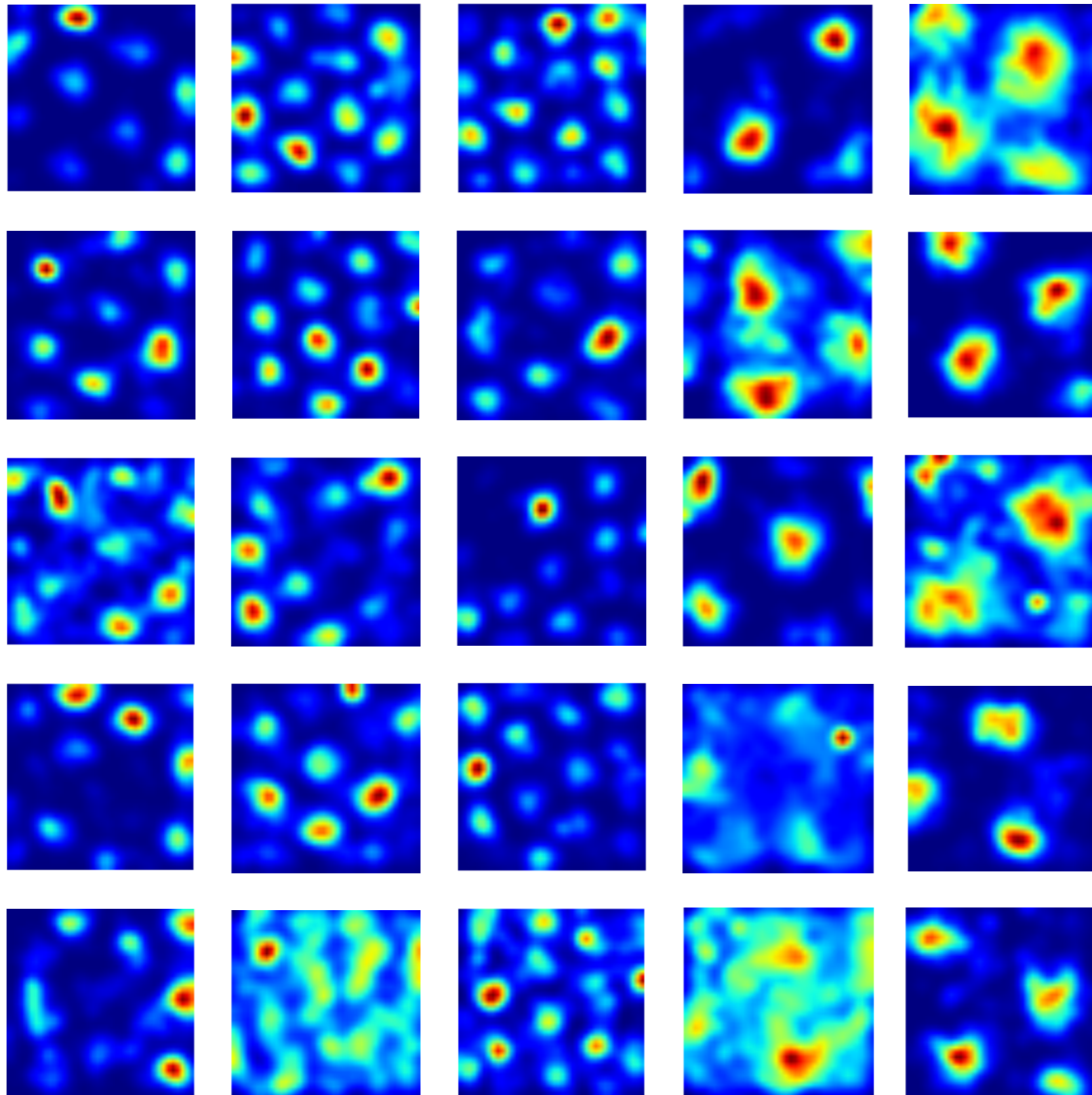
b Mean grid orientation: -20°



Avg rate: 1 Hz Peak rate: 7 Hz

Stensola et al, 2015

But... you get mixed bag of “modules” and other neurons (head direction, border, etc.)



also, they are not “on” always

Stensola et al., 2013

$$\int_{-\infty}^{\infty} e^{i\pi x^2} dx = \sqrt{\frac{1}{i}} \lim_{\epsilon \rightarrow 1^-} \sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} a_n$$
$$\frac{\partial_t u + u \partial_x u + \partial_x^3 u = 0}{\varphi(s+t) = \frac{\varphi(s)\sqrt{1-\varphi(t)^4} + \varphi(t)\sqrt{1-\varphi(s)^4}}{1+\varphi(s)^2\varphi(t)^2}}$$
$$\int_0^1 \frac{1}{\sqrt{1-z^4}} dz = \frac{\Gamma(1/4)^2}{2\sqrt{\pi}}$$
$$u_t - u_{xxt} + 3uu_x - 2u_x u_{xx} - uu_{xxx} = 0$$

Department of Mathematical Sciences



Kavli Institute for Systems Neuroscience
Centre for Neural Computation

PEOPLE:

Nils Baas (Math, NTNU and IAS, Princeton)

Gard Spreemann (EPFL)

Magnus Bakke Botnan (TU München)

Erik Rybakken (Math, NTNU)

Future plans